Fostering Mathematical Creativity Through Multiple Solution Tasks

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Abstract- *Fuchs &Gross in 2009, stated that mathematics is one of the primary competencies in everyday life and ranges from simple tasks from counting to simple calculations, forming the bases and foundation of the nation. Over the last decades the needs of the society have changed, and hence the demand for skilled and educated youth possessing 21st-century skills i.e. the 4C's (Creativity, Collaboration, Critical Thinking, Communication) has also mushroomed over the past decade. To overpower such, our education system is experiencing a paradigm shift in the method of teaching and learning pedagogies especially with a core like mathematics which forms the bases of conceptualizing and understanding the application and mechanism of world principles and theories. Therefore, core topics such as mensuration, number system, geometry, data handling, and algebra have become the components of learning and priority at earlier stages of childhood. And hence, different and distinguishable pedagogies of the same is explored as one of the most vivid topics in the present community of mathematics education. In 2002, Owen & Fuchs have claimed that visual representations of mathematical concepts result in better gains under certain conditions, hence giving a better opportunity to students with learning abstract and abstruse concepts of mathematics. This paper aims to explore different pedagogies of mathematics teaching and learning through the use of visualizing aids i.e. manipulatives to form a concrete relation with abstraction in mathematics to a physical or real-life association. Over the period of time strategies developed like STAR (Stimulate- Translate—Apply—Relate), IDDEA(Identify—Discuss—Demonstrate—Emphasise—Assess), VC^2(Visualize—Compare—Correlate), aims to break the steps of teaching into a framework, allowing teachers/educator to use and apply different approaches of teaching i.e. visualization of topic through suitable manipulatives effectively. Allowing the teacher to inculcate the 21st-century skills in their learners.*

Key Words: *Manipulatives, Pedagogy, Framework, teaching method, Mathematics, 21st-century skills, teaching strategies, conceptualization, visualization*

Introduction

Multiple solutions to the problem consist of employing multiple methods that lead to multiple outcomes. Therefore, it is considered as a good pedagogical practice for promoting the students'

habit of exploring more than one outcome and using more than one method for solving the problem by using their creative potential. In this article, it is discussed that multiple solution tasks approach provides the opportunities to the learner to enhance their capacity with respect to the dimensions of mathematical creativity namely fluency, flexibility and originality. Therefore, different multiple solutions tasks, problems and activities from real-life setting are presented to foster the mathematical creativity of middle school children.

In the 21st century, every country has a great need for creative talents to maintain its own existence and prosperity in the competitive world. Therefore, every nation emphasizes to produce gray gold rather than black gold. For which every nation requires creative mathematicians, creative scientist and creative doctors, etc. The nature of mathematics provides a suitable platform for developing creativity. It plays an important role in mathematics learning and considered as a driving force behind all of the innovations. Expertise in science and technology also requires higher-order thinking skills in the field of mathematics; ergo the world is becoming more and more inclined towards mathematics and science. Due to this indispensable role, mathematics has become an integral part of every new innovation. NPE (1986) has envisaged that "mathematics should be visualized as the vehicle of communication to train a child to think, to reason, to articulate and to analyses logically. It should be treated as a concomitant of any subject involving analysis and synthesis". In addition, NCF (2005) also emphasized for developing children's abilities of mathematisation. Therefore, one of the goals of the Indian education system is to foster creativity among learners. Which use of open-ended problems and innovative practices in mathematics teaching could stimulate creativity, original thinking and innovation in mathematics and science? In contrast, traditional methods of teaching and assessment focus only on lower-order thinking skills. Notwithstanding, many children dislike mathematics and showed several reasons for this happening, in which most mentioned one is that mathematics is a rigid, difficult, dry, boring, irrelevant formulae and taught in the classroom without connecting the real-life setting. Besides above, some mystical cases were also reported in the history of mathematics education where insight came directly from God in exceptional cases like Blaise Pascal, Shakuntla Devi and Srinivasa Ramanujan, etc.

Mathematical Creativity

However, several research studies have already shown the importance of mathematical creativity like one of the greatest assets of a nation (Singh, 1985), plays a vital role in the full cycle of advanced mathematical thinking (Ervynck, 1991), ensures the growth of the field of mathematics as a whole (Sriraman, 2004), therefore, it is considered the major component in the education of the 21st century (Mann, 2005). Hadamard (1945) influenced by Gestalt psychology delineated the creative process- preparation-incubation-illumination and verification. Notwithstanding, there is a lack of an acceptable definition of mathematical creativity (Mann, 2006). Haylock (1997) writes that there is no single definition of creativity that is generally accepted or used in researches and books. There are at least two major ways in which the term is used: (i) thinking that is divergent and overcomes fixation and (ii) the thinking behind a product that is perceived

as impressive by a large group of people, e.g. work of arts. But in the present study mathematical creativity is defined as a process of generating new/significant ideas making theoretical ideas practical, converting innovative ideas of other fields into the field. It is the process of formulating hypotheses in the mathematical situations, testing and retesting these hypotheses and modifying and retesting again and so on (Singh, 1985). Mathematical creativity includes three main dimensions namely fluency, flexibility, and novelty. *Fluency* is the ability to generate different responses where the quantity of right, relevant and unrepeated response and not quality of response is emphasized. *Flexibility* is the ability to produce a diversity of ideas of categories/approach to solve mathematical questions that are relatively unstructured and categorized. *Originality* is the ability to produce remotely associated or uncommon responses.

Hadamard (1945) along with prominent mathematicians George Birkhoff and George Polya and eminent scientists Einstein undertook a study to know about the mental images used in doing mathematics: Neurologists, scientists, and mathematicians reached on the conclusion that mathematical problems of synthesis nature are processed by the right hemisphere of the brain and activation in balance form of right and left both the hemispheres are necessary to solve the problems of mathematical creativity. Left brain system only solves problems of the lower level of objectives. When brain solves the synthesis type problems of mathematics, the problem passes through four stages of mathematical thinking i.e., *Preparation*- concentration and deep involvement in a problem to collects necessary information and facts (i.e. what is given and what is to find out in this problem?) for solving the problem. This is followed by a second stage-*Incubation*- in the stage, the problem solver works hard continuously and if a problem is not solved and one felt exhaust mentally the problem is set aside for while but work on a solution continues at the unconsciously. This stage is very important because after fatigue the problem solver is either to relax or to recover. When work on problem at the unconscious mind is going on the third stage-*Illumination* or insight occurs. It should be clear here that insight is driving force required to move towards a foundation of new knowledge i.e. creativity. This is the stage when creative solution occurs and the problem solver feels pleasure. But the process of thinking to finalize creative product is not ended yet. In the next stage i.e., *Verification* elaboration and refinement of the insightful solution take place. After verifying the result, one formulates hypotheses concerning cause and effect relationship, test-retest the hypotheses and finally try to reach on final output (model developed by Gestalt, 1926 and adopted by Hadamard, 1945). or the development of mathematical creativity, the activities are performed by the right brain hemisphere for incubation and insight stages and rest two stages by the left hemisphere, therefore, interaction between these two hemispheres should be in the balanced form. These four stages have shown the characteristics of the mathematician's creative process.

As it is evident from the various literatures several methods and techniques are used for delivering the contents in the classroom for achieving the desired learning outcomes. One can use a lecture method for achieving the objective of memorization level, demonstration method for comprehensive level and constructivist approach (problem-solving method, project method,

heuristic method and brainstorming technique) for developing the creative thinking among the students. According to Hashimoto there are two methods of fostering mathematical creativity. First, the open-ended approach which means end products is open. It means the first incomplete problem should be presented in front of students, and welcomes the different responses of the student to solve the problem by using their knowledge, skills and way of thinking. Another method is 'From problem to problem' which provides opportunities for the students to formulate new problems by using generalization, analogy, and the idea of converse.

Review of Literature

The findings of the previous cross-sectional studies have already shown that mathematical creativity was found to be significantly related with problem-solving performance (Singh, 1993; Tyagi, 2015); achievement in mathematics (Sak and Maker, 2006; Tuli, 1979); self-concept in mathematics (Singh, 2000); attitude towards mathematics (Tuli, 1979; Singh, 2000); mathematical aptitude (Tyagi, 2014; Jensen, 1973; Tuli, 1979); mathematical intelligence (Tyagi, 2017); personality characteristics (Singh, 1988); emotional intelligence (Jhony, 2008). Thus, the review of research studies and literature reveals significant points: (1)Most of the research studies have been conducted to find out the relationship between mathematical creativity and mathematical aptitude, mathematical problem-solving performance, etc. and only theoretical ideas are given and (2) very few attempts have been made to identify the cause and effect relationship between mathematical creativity and other variables (which one is the independent variable and dependent variable and vice-versa) and how to foster mathematical creativity among middle school children. Therefore, in this article, some examples/activities are suggested to use for enhancing divergent thinking among students.

Activities: Students solve the problems with divergent point of view

Suggest to the students for the following tasks:

1. By using 4 four times you have to make the first three whole number, prime number, for which, you are fully free to use the mathematical operations (+, -, × and ÷) and rules, formulae, etc.

Examples of some responses

A. Use of simple operations

(i) $4 - 4 + 4 - 4 = 0$

(ii) $4 + 4 - 4 - 4 = 0$

(iii) $4 \times 4 - 4 \times 4 = 0$

(iv) $\frac{4}{4} - \frac{4}{4} = 0$

B. Use of BODMAS formula

(i) $4 \div 4 \times 4 - 4 = 0$

(ii) $[4 - \{4 - (4 - 4)\}] = 0$

(iii) $[(4 - 4) - (4 - 4)] = 0$

(iv) $[(4 + 4) - (4 + 4)] = 0$

C. Use of decimal numbers

(i) $4.4 - 4.4 = 0$

(ii) $0.4 - 0.4 + 0.4 - 0.4 = 0$

(iii) $\frac{0.4}{0.4} - \frac{0.4}{0.4} = 0$

(iv) $0.4 \times 0.4 - 0.4 \times 4 = 0$

D. Use of exponential form

(i) $(4)^0 + (4)^0 - (4)^0 - (4)^0 = 0$

(ii) $(4)^4 - (4)^4 = 0$

(iii) $(\sqrt[4]{4}) - (\sqrt[4]{4}) = 0$

(iv) $(\sqrt[2]{4} \div \sqrt[2]{4}) - (\sqrt[2]{4} \div \sqrt[2]{4}) = 0$

(v) $\{(4)^0 \times (4)^0\} - \{(4)^0 \times (4)^0\} = 0$

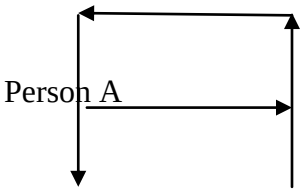
(vi) $\frac{\sqrt[3]{4}}{\sqrt[3]{4}} - \frac{\sqrt[5]{4}}{\sqrt[5]{4}} = 0$

E. Use of logarithms

(i) Log 4 + Log 4 – Log 4 – Log 4 = 0

(ii) $Log(4)^4 - Log(4)^4 = 0$

F. Use of distance and direction



Distance of Person A from starting point to destination = 0 Km

Now you have to use divergent approach for 1, 2 and 3 by using 4 four times.

2. How to make 9 by using 3?

Examples of some responses

(i) $3 + 3 + 3 + 3 = 9$

(ii) $3 – (–3 – 3) = 9$

(iii) $(3)^2 = 9$

(iv) $(3)^0 \times 3 \times 3 \times (3)^0 = 9$

(v) $(\sqrt[2]{3}) \times (\sqrt[2]{3}) \times (\sqrt[2]{3}) \times (\sqrt[2]{3}) = 9$

Now you have to use divergent approach for the square number like 16, 25 and so on.

3. Find out the square numbers which will be equal to four times the square of another numbers.

Examples of some responses

(i) $4 or (2)^2 = 4 \times 1^2$

(4, 1) (64, 16) (1024, 256) ---------- using numbers

(ii) $(1) or (1)^2 = 4\left(\frac{1}{2}\right)^2$

$\left(1, \frac{1}{2}\right)\left(\frac{1}{16}, \frac{1}{64}\right)\left(\frac{1}{256}, \frac{1}{1024}\right)$---------- using fractions

(iii) $0.04 or (0.2)^2 = 4 \times (0.1)^2$

(0.04, 0.01) (0.64, 0.16) (10.24, 2.56) ---------- using decimal numbers

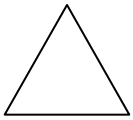
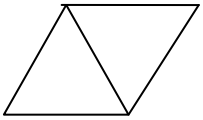
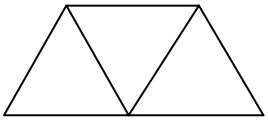
(iv) $4 or (2\sqrt{2})^2 = 4 \times (\sqrt{2})^2$

$\left(2\sqrt{2}, \sqrt{2}\right)$ $(4\sqrt{2}, {}_{2\sqrt{2}})$ $(8\sqrt{2}, {}_{4\sqrt{2}})$ ---------- using exponential form

Now you have to find out the square numbers which will be equal to three times of the other square numbers.

4. Triangles are made by using matchsticks as shown in the following figures:

For example

				
(1)	**(2)**	**(3)**	**(4)**	**(5)**

When the number of triangles is 6, how many match sticks will be required?

5. Find out the similarities and differences between the numbers of 169 and 961.

Examples of some responses

6. Find out the similarities and differences between the square and rectangle.

Examples of some responses

7. The sum of consecutive odd numbers resulting in a full square $1 = 1^2$, $1 + 3 = 2^2$, $1 + 3 + 5 = 3^2$, etc. Similarly find out the sequence for the sum of consecutive even numbers in a full square

Examples of some responses

Concluding Remark

During the school internship programme I observed that most of the teachers use close-ended problems in mathematics learning which is only limited to cramming the formula, understanding for solving the problems and able to achieve only memorization and understanding level (superficially), but fail to achieve a higher level of objectives of cognitive domain i.e. creating. Multiple solution tasks approach is based on constructivist approach through which learner gets the opportunity to think out-of-the-box and indulge in active thinking and try to related mathematics with the real-life settings and reflects mathematization of mind. Mathematical puzzles and games, etc. help not only developing a positive attitude towards mathematics but also help for making connections between mathematics and everyday life. In this way, we can train our young minds for divergent thinking as potential factors of creativity in mathematics.

Mathematical creativity is generally related to problem-solving and or problem-posing at the school level. Therefore, students should be provided opportunities to engage in struggling to solve mathematics problem which is ill-defined or open-ended. By which students will be encouraged to reflect on their own ideas. In addition, creative environments and creative teachers both are very important factors to foster mathematical learning at levels of schooling. Therefore, this is a thrust area to train teachers and making them aware of the characteristics of creative thinking and environments is one of the necessities that one should consider. Ergo, a support system (learning material) will be developed to enhance the creative thinking of students.

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