Comparison of Secondary Geometry Syllabus Being Taught in India and the United States of America

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Abstract- Mathematics is plays an important role in human life. It is a tool which helps us to understand the world around us and with the knowledge of math humans make the world a better place to live. Geometry is one of the most important branches of mathematics which helps us to visualise different relations and wonderful properties of figures and shapes. It helps to critically analyse the given information and retrieve visual pattern and calculate some concrete information about that figure and shape. It teaches the inductive and deductive reasoning to form definitions, axioms and theorems. Geometry starts from conception of a human life till its death. It is involved in every aspect of human life. A doctor checks the dimension of an embryo and compares it with a healthy embryo's size. The moment a child is born Geometry enters into its life automatically. Doctors use Geometry to decide if it's a healthy baby. Within a few months' baby learns to identify different shapes and objects and learns the meaning of it. As the child grows older the whole life starts revolving around Geometry. Geometry is used in architecture to design buildings' shape and their blueprints. Robotics, and video games use Geometry to simulate 3D images in virtual world. GPS uses geometry to give the correct location. Geography uses Geometry to calculate the location, trajectories of different stars. Medical profession uses it for CT scans, MRIs for diagnosis and treatment of different diseases. Educationist use it to increase student achievement. Real world used a lot of Geometry to decide angles between the blades of a fan, track length of a circular ground, how much paint is needed for a wall, what should be the angle of a ramp to make it easier for disabled person, translation in a game of chess, dilation while making copies or printouts, rotation in revolving door, reflection in symmetric images, making miniatures of bigger objects, using congruent and similar triangles while constructing homes, line and segments that intersect circles in bicycle chain. The current paper is an attempt to find the topics covered in Geometry in two different countries. In this paper researchers tried to explore the syllabus of Geometry being taught at secondary level in two different countries- India and the United States. This paper explored the similarities and differences in Geometry taught at secondary level in these countries. Findings suggested some significant differences and a lot of similarities in topics being taught. This was a qualitative study to compare and figure out the quality of Geometry being taught.

Key Words: Geometry, Course Overview, Curriculum, Secondary School, Countries

Introduction

Mathematics play an important role in human life. It is a tool which helps us to understand the world around us and with the knowledge of math humans make the world a better place to live. Geometry is one of the most important branches of mathematics which helps us to visualise different relations and wonderful properties of figures and shapes. It helps to critically analyse the given information and retrieve visual pattern and calculate some concrete information about that figure and shape. It teaches the inductive and deductive reasoning to form definitions, axioms and theorems. Geometry starts from conception of a human life till its death. It is involved in every aspect of human life. A doctor checks the dimension of an embryo and compares it with a healthy embryo's size. The moment a child is born Geometry enters into its life automatically. Doctors use Geometry to decide if it's a healthy baby. Within a few months baby learns to identify different shapes and objects and learns the meaning of it. As the child grows older the whole life starts revolving around Geometry. Geometry is used in architecture to design buildings' shape and their blueprints. Robotics, and video games use Geometry to simulate 3D images in virtual world. GPS uses geometry to give the correct location. Geography uses Geometry to calculate the location, trajectories of different stars. Medical profession uses it for CT scans, MRIs for diagnosis and treatment of different diseases. Educationists use it to increase students achievement. Real world used a lot of Geometry to decide angles between the blades of a fan, track length of a circular ground, how much paint is needed for a wall, what should be the angle of a ramp to make it easier for disabled person, translation in a game of chess, dilation while making copies or printouts, rotation in revolving door, reflection in symmetric images, making miniatures of bigger objects, using congruent and similar triangles while constructing homes, line and segments that intersect circles in bicycle chain. The current paper is an attempt to find the topics covered in Geometry in two different countries and compare them.

People think that mathematics is taught in the same way all over the world. Indeed, end outcome is same everywhere, only the difference is in the organisation of curriculum at different grade levels and how it is being taught. Minute differences of math language and topics being taught also make the difference in math education in different countries.

Objectives

- To understand the mathematics education system in India and US
- To study the aims of mathematics education in India and US
- To study the similarities in Geometry taught at secondary level in India and US
- To find out the differences in Geometry taught at secondary level in India and US
- To explore the potential of the curriculum of mathematics education in India and US

Methodology
The purpose of the present study was to compare curriculum of two countries. After reviewing existing literature and different research approaches, we found qualitative approach most suitable to achieve the objectives of the present study. Document analysis was done to study the aims of

mathematics education as well as to study the curriculum of two countries in terms of similarities and differences.

Documents for the present study

- Syllabus for Secondary and Higher Secondary Classes, NCERT
- Mathematics textbook for classes 9 and 10, NCERT
- California Common Core State Standards, adopted by the California State Board of Education, USA
- Big Ideas Math for Geometry by Ron Larson and Laurie Boswell

Analysis of Data
Content analysis was done for the comparison of documents.

Comparison of Education System of the USA and India

Primary and Secondary Schools in the USA
In United States schools are organized into 3 levels, elementary (primary) schools, middle schools, and high (secondary) schools. Elementary serves from kindergarten to grade 5-6, middle school serves grade 6-7 to 8 and high school serves from grade 9-12 depending on state and school district policy. There are nicknames for different grade levels- 9th graders are called Freshman, 10th grader is called Sophomores, 11 graders are called Juniors, and 12th graders are called Seniors. There Are two kinds of schools- Public (called government schools in India) and Private (called public schools in India). Also, the students can only go to their neighborhood school, public school (government schools called in India). All three levels have their school boundary map. A state has different counties and a county can have several school districts. A school district runs all the elementary, middle and high school of that city. Based on the part of the city there is an assigned elementary, middle and high school. The Second authore teachers in Fremont Unified School District which is part of Alameda county and our country is a part of county California. Fremont Unified school district has 29 Elementary, 5 Middle and 5 high schools. All schools in the same district follow the same textbooks, rules, and calendar but they can have their own bell schedule.

California Common Core State Standards for Mathematics (CA CCSSM) has three main Principles- focus, coherence, and rigour. These three principles not only prepare all California students for college, careers, and civic life but also helps students to grow as a mathematically competent person. The goal of these standards is to prepare an individual who would not only be able to make wise decisions in their life, but would also be able to add some value to the world. World-class education for all students, supporting lifelong learning, narrowing the achievement gap, and helping students develop the skills and knowledge necessary to fully participate in the global economy of the twenty-first century are the main commitments of California's implementation of the CA CCSSM.

Table:1 Guiding Principles for Mathematics Programs in California

Table IN-1. Guiding Principles for Mathematics Programs in California

Guiding Principle 1: Learning
Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.

Guiding Principle 2: Teaching
An effective mathematics program is based on a carefully designed set of content standards that are clear and specific, focused, and articulated over time as a coherent sequence.

Guiding Principle 3: Technology
Technology is an essential tool that should be used strategically in mathematics education.

Guiding Principle 4: Equity
All students should have a high-quality mathematics program that prepares them for college and careers.

Guiding Principle 5: Assessment
Assessment of student learning in mathematics should take many forms to inform instruction and learning.

United States Math curriculum is not integrated. It is different than India. They study on subject whole year and they can choose the path according to their potential and interest. (Fremont Unified School District) follows the following pathway. There are 4 different pathways and a student has to earn a specific grade to enter the accelerated one and they need to maintain a specific grade in order to stay in the pathway. If they perform below a certain criterion, then they are sent back to lower level math class.
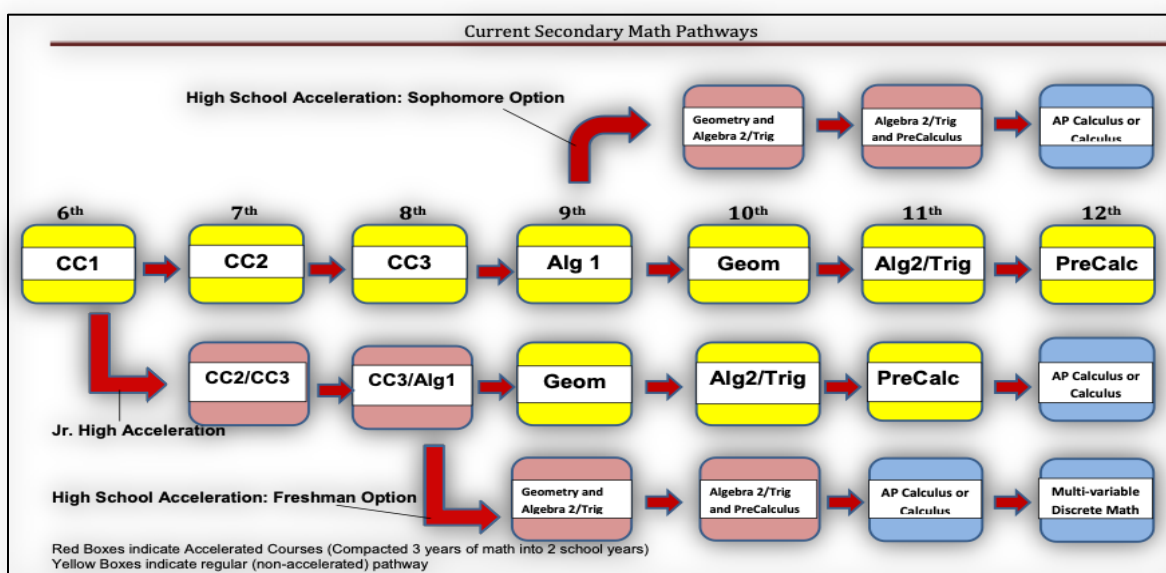

Figure : 1 Current Secodary Maths Pathways

For secondary level according to A-G Subject requirements a student has to take 3 years of math, but 4 year is recommended. It means they need to pass 3 years of equivalent of math in order to graduate from high school. So, A student needs to do at least up to Algebra 2/Trig

course in order to have a high school diploma. If a student is really excellent and on an accelerated path then can learn up to Multivariable calculus/Discrete Math.

First author teaches classes of Geometry. Geometry taught at high school is centered around plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Following are the standards(objectives) of Geometry for California.

Congruence

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions
- Similarity, Right Triangles, and Trigonometry
- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles
- Circles
- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles
- Expressing Geometric Properties with Equations
- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically
- Geometric Measurement and Dimension
- Explain volume formulas and use them to solve problems
- Visualise relationships between two-dimensional and three-dimensional objects
- Modeling with Geometry
- Apply geometric concepts in modeling situations
- Conditional Probability and the Rules of Probability
- Understanding dependence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform
- probability model.
- Using Probability to Make Decisions
- Use probability to evaluate outcomes of decisions.

Every state has their own standards and the different school districts to choose their own books to cover those standards. The second author teaches in the Fremont Unified School district of California, and this district follows Big Ideas Math for Geometry by Ron Larson and Laurie Boswell. Following is the syllabus covered by the book.

Basics of Geometry

- Points, Lines, and Planes
- Measuring and Constructing Segments
- Using Midpoint Formulas
- Perimeter and Area in the Coordinate Plane
- Measuring and Constructing Angle
- Measuring and Constructing Angle

Reasoning and proofs

- Conditional Statements
- Inductive and Deductive Reasoning
- Postulates and Diagrams
- Algebraic Reasoning
- Proving Statements about Segments and Angles Explorations
- Proving Geometric Relationships

Parallel and Perpendicular Lines

- Pairs of Lines and Angles
- Parallel Lines and Transversals
- Proofs with Parallel Lines
- Proofs with Perpendicular Lines
- Slopes of Lines
- Equations of Parallel and Perpendicular Lines

Transformations

- Translations
- Reflections
- Rotations
- Congruence and Transformations
- Dilations
- Similarity and Transformations

Congruent Triangles

- Angles of Triangles
- Congruent Polygons
- Proving Triangle Congruence by SAS

- Equilateral and Isosceles Triangles
- Proving Triangle Congruence by SSS
- Proving Triangle Congruence by ASA and AAS
- Using Congruent Triangles
- Coordinate Proofs

Relationships within Triangles

- Perpendicular and Angle Bisectors
- Bisectors of Triangles
- Medians and Altitudes of Triangles
- The Triangle Midsegment Theorem
- Indirect Proof and Inequalities in One Triangle Explorations
- Inequalities in Two Triangles

Quadrilaterals and Other Polygons

- Angles of Polygons
- Properties of Parallelograms
- Proving That a Quadrilateral is a Parallelogram Explorations
- Properties of Special Parallelograms
- Properties of Trapezoids and Kites

Similarity

- Similar Polygons
- Proving Triangle Similarity by AA
- Proving Triangle Similarity by SSS and SAS
- Proportionality Theorems

Right Triangles and Trigonometry

- The Pythagorean Theorem
- Special Right Triangles
- Similar Right Triangles
- The Tangent Ratio
- The Sine and Cosine Ratios
- Solving Right Triangles
- Law of Sines and Law of Cosines

Circles

- Lines and Segments That Intersect Circles

- Finding Arc Measures
- Using Chords
- Inscribed Angles and Polygons
- Angle Relationships in Circles
- Segment Relationships in Circles
- Circles in the Coordinate Plane

Circumference, Area, and Volume

- Circumference and Arc Length
- Areas of Circles and Sectors
- Areas of Polygons
- Three-Dimensional Figures
- Volumes of Prisms and Cylinders
- Volumes of Pyramids
- Surface Areas and volumes of Cones

Probability

- Sample Spaces and Probability
- Independent and Dependent Events
- Two-Way Tables and Probability
- Probability of Disjoint and Overlapping Events Explorations.
- Permutations and Combinations
- Binomial Distributions

Mathematics Education in India

"The main goal of mathematics education in schools is the mathematization of the child's thinking. Clarity of thought and pursuing assumptions to logical conclusions is central to the mathematical enterprise. There are many ways of thinking, and the kind of thinking one learns in mathematics is an ability to handle abstractions, and an approach to problem solving". Teaching of Mathematics, NCERT 2005.

In India, schooling is free and compulsory for every child and mathematics is a compulsory subject of study which has important implications for mathematics curriculum. Access to quality mathematics education is a right of every child. Mathematics education after the recommendations of several commissions and committees becomes affordable and enjoyable at school level. The National Curriculum Framework 2005 (NCF) is developed by NCERT. Vision of NCF for excellent mathematical education is based on the twin premises that all students can learn mathematics and that all students need to learn mathematics. This vision helps in achieving the goals of Universalisation of schooling in India. Mathematics education up to class 8 helps children to be ready for the challenges that they face in life. Mathematics education

till class 10 is based on utilitarian principles. It connects mathematics to real life of the student. Student can feel mathematics in daily life and connect herself or himself to mathematics education at school level. She or he enjoys mathematics and develops the ability of *Mathematics action* which is the ultimate aim of mathematics education. In recent decades mathematics education has undergone many changes. This change was in terms of aims of mathematics education.

Structure of School Education

"Article 45 of the Constitution of **India** is a directive principle and is a provision for free and compulsory education for children. The State shall endeavor to provide, within a period of ten years from the commencement of this **Constitution**, for free and compulsory education for all children until they complete the age of fourteen years." Indian Constitution Mathematics is an important school subject and its compulsory to learn up to class 10. Indian education commission 1964-66 made several recommendations including mathematics as a compulsory subject till class 10. In India we provide free and compulsory education to a child up to the age of 14. Education is a right of every child in India. Right to education Act 2009 (RTE) is an Act of the Parliament of India, which empower every child to get free and compulsory education between the ages of 6 - 14 in India under Article 21 a of the Indian constitution.

The structure of education includes primary level which covers classes 1-5, upper primary education which covers classes 6-8, secondary level which includes classes 9-10.

Primary Level – Ages: 5-9 years /classes: 1-5
Subjects: Hindi, Urdu, English, EVS (3-5), Mathematics

Syllabus of mathematics: **Geometry**, Numbers, Number Operation, Money, Measurement of Length, weight, Volume and Time, Data Handling, Patterns.

Upper Primary Level-Ages: 10-12 years/Classes: 6-8
Subjects: Languages, Science, Social Science and Mathematics

Syllabus of Mathematics: Number System, Algebra, Ratio and Proportion, **Geometry**, Mensuration, Data Handling

Primary level and upper primary level comprise the elementary education in India.

Secondary Level - Ages: 13-14 years/Classes: 9-10
Subjects: Languages, Science, Social Science, Mathematics

Syllabus of mathematics in India
Class 9- Units

- Number Systems
- Algebra

- Coordinate Geometry
- Geometry
- Mensuration
- Statistics and Probability

Appendix:

- Proofs in Mathematics,
- Introduction to Mathematical Modelling.

Geometry

Introduction to Euclid's Geometry

History – Euclid and geometry in India. Euclid's method of formalizing observed phenomenon into rigorous mathematics with definitions, common/obvious notions, axioms/postulates, and theorems. The five postulates of Euclid. Equivalent versions of the fifth postulate. Showing the relationship between axiom and theorem".

> "Given two distinct points, there exists one and only one line through them".
> "Two distinct lines cannot have more than one point in common".

Lines and Angles

- If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and the converse.
- If two lines intersect, vertically opposite angles are equal.
- Results on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines.
- Lines, which are parallel to a given line, are parallel.
- The sum of the angles of a triangle is 180°.
- If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

Triangles

- "Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence)".
- "Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence)".
- "Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS Congruence)".
- "Two right triangles are congruent if the hypotenuse and a side of one triangle are equal (respectively) to the hypotenuse and a side of the other triangle".
- "The angles opposite to equal sides of a triangle are equal".

- "The sides opposite to equal angles of a triangle are equal".
- "Triangle inequalities and relation between 'angle and facing side'; inequalities in a triangle".

Quadrilaterals

- "The diagonal divides a parallelogram into two congruent triangles".
- "In a parallelogram opposite sides are equal and conversely".
- "In a parallelogram opposite angles are equal and conversely".
- "A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal".
- "In a parallelogram, the diagonals bisect each other and conversely".
- "In a triangle, the line segment joining the mid points of any two sides is parallel to the third side and its converse".

Area

- Review concept of area, recall the area of a rectangle
- Parallelograms on the same base and between the same parallels have the same area
- Triangles on the same base and between the same parallels are equal in area and its converse.

Circles

- "Through examples, arrive at definitions of circle related concepts, radius, circumference, diameter, chord, arc, subtended angle".
- "Equal chords of a circle subtend equal angles at the center and (motivate) its converse".
- "The perpendicular from the center of a circle to a chord bisects the chord and conversely, the line drawn through the center of a circle to bisect a chord is perpendicular to the chord". "There is one and only one circle passing through three given non-collinear points".
- "Equal chords of a circle (or of congruent circles) are equidistant from the center(s) and conversely".
- "The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle".
- "Angles in the same segment of a circle are equal".
- "If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, the four points lie on a circle".
- "The sum of either pair of opposite angles of a cyclic quadrilateral is 180° and its converse".

Constructions

- "Construction of bisectors of a line segment and angle, 60°, 90°, 45° angles etc., equilateral triangles".
- "Construction of a triangle given its base, sum/difference of the other two sides and one base angle".
- "Construction of a triangle of given perimeter and base angles".

Class 10 - Units

- Number Systems
- Algebra
- Trigonometry
- Coordinate Geometry
- Geometry
- Mensuration
- Statistics and Probability".

Appendix:

- Proofs in Mathematics
- Mathematical Modelling".

Geometry
Triangles
Definitions, examples, counterexamples of similar triangles.

- "If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio".
- "If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side".
- " If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
- "If the corresponding sides of two triangles are proportional, their corresponding angles are equal, and the two triangles are similar.
- "If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar".
- "If a perpendicular is drawn from the vertex of the right angle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other".
- "The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides".

- "In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides".
- "In a triangle, if the square on one side is equal to the sum of the squares on the other two sides, the angles opposite to the first side is a right triangle".

Circles

Tangents to a circle motivated by chords drawn from points coming closer and closer to the point.

- "The tangent at any point of a circle is perpendicular to the radius through the point of contact".
- "The lengths of tangents drawn from an external point to a circle are equal".
- Constructions
- "Division of a line segment in a given ratio (internally)".
- "Tangent to a circle from a point outside it".
- "Construction of a triangle similar to a given triangle".

Comparison of Mathematics Education with respect to Geometry Syllabus in India and the United States of America

The big picture of syllabi for both countries is the same. There are minute differences about a few topics, language of math and structures of math courses at different levels of grades. After analysing the syllabus, the authors found out the major difference in syllabi are a topic of Transformation and some smalled concepts which are present in the USA's Geometry curriculum. Transformation widely covers the details about translation, reflection, rotation and dilation. These concepts are explained mostly in context of cartesian coordinates. For Inductive/deductive reasoning law of detachment, law of syllogism and contrapositive are the other concepts included. Students are being taught to use Algebraic properties of equalities to solve an equation whereas in India it is direct calculation without applying the law. Concept of two column proof, paragraph proof and flow proof are introduced. Properties of congruence and equality is introduced, To write measurement of any angle m∠A notation is used instead of ∠A.

Conclusion

Mathematics is a universal subject. Mathematics is an essential part of school curriculum. In world, every organisation gave it an important place in the curriculum. Despite being universal subject there are few differences in the syllabus of India and USA. This difference is not in terms of concepts. This difference is because of the approach and structure of school education system. For example, in India, students learn integrated math. All students learn geometry in each class along with other themes such as algebra, number system, probability and statistics, however, in the USA, students learn just one math content or the combination of two for the whole year depending on path opted.

Assessment of learning is different in two countries. In India if a student is not able to learn geometry but he or she is good at other themes such as number system, algebra, etc. he or she may pass the exam and goes in next class. Students do not get any chance to learn concepts of geometry again in next class. Curriculum for upper grade/ class is built on the curriculum of lower grade / class. However, in the USA, if a student fails to complete the geometry course, he or she needs to learn it again to attain certain credits for high school graduation.

Curriculum of India is as per government and same textbook is adapted all over the country in different languages whereas in USA curriculum strategy allows different states to decide their own standards and different counties (parts of the state) choose their own textbooks covering those standards. Looking at the structural difference we can say we may adapt some portion of USA school curriculum of geometry in India as we need to compete PISA Programme for Intermation Student Assement and other international assessment surveys.

References

https://www.cde.ca.gov/ci/ma/cf/

https://www.cde.ca.gov/ci/ma/cf/documents/mathfwgeometryjl.pdf

https://www.cde.ca.gov/be/st/ss/documents/ccssmathstandardaug2013.pdf

http://righttoeducation.in/know-your-rte/about

Teaching of mathematics, position paper 2005, NCERT

National Curriculum Framework 2005 NCERT New Delhi

Syllabus of secondary and higher secondary, NCERT

French, D. (2007) Teaching and Learning Geometry, Continuum, New York.

Driscoll, P, Lambirth, A. and Roden, J. (2015) The Primary Curriculum (second edition), Sage, New Delhi.