Learning Outcomes in Geometry at Secondary Stage

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Abstract- *Geometry is recognized as not only one of the most important components of the school mathematics curriculum but also, alongside algebra, as one of the most important elements of mathematics itself. The reasons for including geometry in the school mathematics curriculum are providing opportunities for learners not only to develop spatial awareness, geometrical intuition and the ability to visualize, but also to develop knowledge and understanding of, and the ability to use, geometrical properties and theorem. In general, the twin concerns of the mathematics curriculum are: What can mathematics education do, to engage the minds of every student and strengthen the student's resources? As mathematics is a compulsory subject at the secondary stage, access to quality mathematics education is the right of every child. So, while talking about quality mathematics education, in general, in what sense "the learning outcome" can be understood? In fact, 'curricular expectations' define what a child should know, be able to do and dispositions that should be acquired over a period of time. Learning outcomes are derived from curricular expectations and syllabus is provided to help all the stakeholders in understanding the goals to be achieved. The learning outcomes are generally treated as assessment standards or benchmarks for assessment. Children are often assessed with paper pencil tests, which include certain types of questions without proper analysis whether these questions have potential to assess child's level of understanding in a particular class. There is a need for a teacher to understand how children progress in learning continuum of mathematics, how children's learning matches with the expectations of curriculum and what pedagogies she/he has to adopt for maximizing child's learning. This article deals with, expectations of the mathematics curriculum at secondary stage, especially for Geometry portion. Here, we try to explore the learning outcomes in Geometry for Secondary Stage Mathematics along with some suggested pedagogical processes, which may be undertaken to achieve the outcomes. These pedagogical processes are not exhaustive. They are suggestive in nature, and may vary according to the learner's context.*

Introduction

Highlighting the end product of the learning process normally leads to achieving it through rote memorisation without understanding. However, the highlighting of the end product in mathematics learning lays emphasis on remembering the facts and using algorithms without understanding. Further it develops a handicap in children about use/applications of mathematical ideas in daily life. The teachers are accepted to provide learning opportunities while transacting different concepts of mathematics to help children explore and connect with their immediate

surroundings, (self, family, school etc.). The suggested pedagogical processes include examples for the same. Learning is a continuous process. The learning outcomes are impacted by the learning /pedagogical processes used to develop competencies. The learners are expected to realise and use mathematics as an important tool that they can talk about, use and explore as well as understand its structure.

Curriculum expectations at secondary stage

The main goal of mathematics education is mathematisation of child's thought processes. At the secondary stage, students begin to perceive the structure of mathematics as a discipline. They become familiar with the characteristics of mathematical communication: carefully defined terms and concepts, the use of symbols to represent them, precisely stated proportions and proofs justifying propositions. These aspects are developed particularly in the area of geometry. Students develop these aspects with algebra, which is important not only in the application of mathematics, but also with in mathematics in providing justifications and proofs. At this stage, students integrate many concepts and skills that they have learnt into a problem-solving skill ability.

Keeping in view the above objectives, in National Curriculum Framework-2005, curricular expectations at secondary stage are as follows:

- To develop ability for mathematisation (ability to think logically, formulate and handle abstractions) rather than knowledge of mathematics(formal and mechanical) procedures.
- To develop ability to pursue assumptions to their logical conclusions and to handle abstractions.
- To develop ability and the attitude to formulate and solve problems.
- To develop ability to address interdisciplinary problems from other domains such as science and social sciences.
- To develop ability of posing and solving problems.
- To develop ability to integrate concepts and skills that they have learnt into a problem-solving ability.
- To develop ability to understand and construct the process involved in mathematical reasoning.
- To develop ability to understand the linkages between mathematics and daily life experiences and across the curriculum.
- To achieve these curriculum expectations in mathematics at secondary stage, a vision has also been framed for learning mathematics at secondary as follows:
- Children learn to enjoy mathematics rather than fear it.
- Children learn important mathematics; mathematics is more than formulas and mechanical procedure.
- Children see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.

- Children pose and solve meaningful problems.
- Children use abstraction to perceive relationship, to see structures, to reason out things, to argue the truths or falsity of statements
- Children understand the basic structure of mathematics. Arithmetic, algebra, geometry and trigonometry, the basic content areas of school mathematics, all offer a methodology for abstraction, relation and generalisation.
- Children understand the basic nature of mathematics such as reasoning, proving, mathematical proofs, communicating, connecting, etc.
- Teachers engage every child in class with the conviction that everyone can learn mathematics towards inclusiveness.

Geometry comprises that branch of mathematics which encourages visual intuition (the most dominant of our sense) to acknowledge the features of theorems, understand proof, perceive reality and give global insight that force a learner to identify the beauty of geometry in surroundings. These are transferable skills that are needed for (but not taught by) all other branches of mathematics. The aims of learning geometry content can be summarized as follows:

- To acknowledge opportunities of geometrical experiences in two and three dimensions;
- To develop knowledge and understanding of and the ability to use geometrical properties and theorems;
- To develop sense and use of conjecture, deductive reasoning and proof;
- To develop skills of applying geometry through problem solving in real world Context; and
- To inculcate a positive attitude towards mathematics

The learners may be provided opportunities in pairs / groups/ individually in the form of proving theorems/problems in geometry. They may be encouraged to generalize the results and provide explanations and justifications for arguments. When a student or a group presents an argument for a proof, others should try to question the validity of that argument. The presenters should try to satisfy them through logic and appropriate results. Some suggested ways are:

Learning

Learning outcomes	**Suggested Pedagogical Processes**
The learner Geometry	**Geometry**
Develops conceptual understanding of axioms and postulates, Cartesian system, lines and angles, triangles, Quadrilaterals, circles etc.	-Motivate discussion among children to find out ways to fix position of a point in A plane. -Activity may be organized based on seating position of each student with Respect to the position of the teachers table or by keeping it between the first rows of student seats, where equal number seats are

	available on each side of it. -While introducing the Cartesian system Of two mutually perpendicular lines in the plane, the concepts of origin, x- axis, y axis, x- coordinate, y-coordinate, position of a point etc. can be explained with the help of this activity. -Either the teacher may draw a diagram of a coordinate plane having a few points With their coordinates on the Board or children may draw it in their notebooks.Then children may be encouraged to discuss about the following: * The number of axes drawn, the number of parts in which these axes divide the plane, the general name assigned to the separts (i.e. quadrants), the way of coding. The placement of points at different places in the plane (i.e. coordinates), the signs of coordinates in different quadrants etc. This may be followed with another activity with the help of graph sheets. -Discussion and thinking can be generated among students about the definitions, axioms and postulates in Euclid's geometry, the concepts of lines and angles, congruence of triangles, quadrilaterals, circles, areas of parallelograms and triangles etc**.** -Students may be given different examples either drawn on Black Board oron a chart paper to familiarize the different concepts such as definitions, Axioms and postulates in Euclid's geometry. Students may be asked to Develop examples to verify these. -Students may be encouraged to cite different examples of quadrilaterals and Circles from real life situations. -GeoGebra based applets on Coordinate Geometry and other concepts of geometry may also be used for discussions
Connects mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines Geometry and number system	Activities may be provided to connect the Map reading skill and the Google map with the help of the concepts of Geometry and number system Algebra and Geometry coordinate geometry. The concepts of lines and rays may be used for explaining the concepts of refraction and reflection in physics. Congruence of geometrical figures can be discussed by the

Algebra and Geometry	students with the help of examples from real life such as photographs, photo copy, coins, currencies, desks, benches, etc.
Communicates in orally and writing the mathematical ideas presented in symbolic, graphical, and pictorial forms	Different activities may be given to the learners to share their ideas in written or oral form based on their understanding of The content. Group discussions can help them by letting them rethink, explain, and Justify their solutions. Following are some of the cases from geometry where students' competency in communications. Can also be assessed. Finds difference between an axiom and a postulate as given by Euclid and explains It to other students. States (without proof) results like: If a ray stands on a line, then the sum of two adjacent angles so formed is 180 and its converse. Students may be encouraged to apply the above results in solving problems in geometry. Students may be asked to state the results on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines, exterior Angle result in written and oral form. Similar activities may be done by the students for other concepts like angle sum property of a quadrilateral, congruence criteria of triangles, properties of parallelogram and its special cases like rhombus, square, rectangle, the properties of the chords of a circle etc.
Uses mathematical reasoning in appropriate situation- Abstract, Spatial, Proportional, Probabilistic, Inductive and Deductive reasoning in Constructing, proving mathematical knowledge and solving problems	In order to think about the proof's students may be encouraged to construct Appropriate figures and discuss with others to arrive at the conclusion as to what is to be proved in each case. During this process they may analyse and state what facts or situations were given and What is to be proved? Let them explore the way(s) to be proceeded for arriving at The final destination with logical reasoning in each stage.

	Following are some of the cases: Student may think about how to apply Euclid's axioms and postulates to prove result like o Two distinct lines cannot have More than one point in common". o vertically apposite angles are Equal. o that on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines -Student may be asked to construct proof for ASA criterion for congruence of triangles by taking SAS criterion as an Axiom. -Student may state and use appropriate logical reasoning to prove that diagonal of a parallelogram divides it into two congruent triangles and other results Related to parallelograms, circles etc. - Students may be encouraged to discuss, analyse and present their arguments about the use of properties of congruent triangles and that of parallelograms to solve problems and prove theorems based on areas of parallelograms and triangles
Demonstrates fluency in mental processes and estimation	On a graph paper coordinate axis may be Drawn on the Black Board. A point with Its coordinates may be marked. One group may be asked to tell a possible position of a point, which is, say, 4 units Away to the left of the given point. The group should estimate such a point, market, tell why they felt so and then calculate The distance. Similar distances may be given in different directions to other Groups. -A list of axioms/postulates/accepted mathematical statements may be provided. Then each group of students should be given a mathematical statement to be proved. The group should estimate which mathematical statement out of the given list can be useful in proving the mathematical statement. They need to present arguments for their claim. Later they can work out

	a proof for the same and check whether they made use of all the mathematical statements they claimed they would use or did they require some more.
Develops visualization skills to assist in processing information, making connections, and solving problems	For example, to prove that '*angles opposite to equal side of an isosceles triangle is equal'* students need to draw an approximate picture of the situation. He/she need to draw an isosceles triangle and then should state what is to be proved as per the figure, what more construction is required etc. While constructing geometrical figures using compasses, they may draw a sketch based on the given information and see for themselves how it helps in processing Information and tell their observations. Discussion can then be held on this. Develops skills in constructing some angles of specific measures, angle bisectors, perpendicular bisectors of line Segment, and triangles with given data. Initially students use visualization to construct an approximate figure and then use standard procedure for constructing
Poses and solves different types of problems	The learner may be asked in pairs groups/ individually to find contexts in mathematics and other subject areas and encouraged to pose and solve different Types of geometrical problems. For example, the theorem *lines which are parallel to the same line are parallel to each other* can be used to solve many Problems from physics and real-life situations using parallel mirrors. Students Need to identify such situations in groups. One group may state an axiom, say, If *equals are added to equals, then the results are equal*. The other group should try to provide example for this statement, like, $4 + 5 = 3+ 67 – 3 = 3 + 1$ So, $4+5+7-3 = 3+6 +3+1$ -Students may be confronted with problem solving situations in areas like congruence of triangles, properties of parallelograms, properties of circles, and chords of a circle etc.
Translates any real-life situation	Discussion may be held to make the students aware about

with the help of a mathematical model	the problems that we encounter in day to day life and how use of geometry can help us overcome that. For e.g. the flooring of rooms of different shapes, plantation to be done in fields of different shapes etc. The learner in pairs / groups/ individually may be encouraged to find which geometrical concept like area of triangle/quadrilateral can be applied. They should solve and interpret it.

Conclusion

This list may be extended to cover entire learning outcomes of Mathematics at secondary stage. Such exemplar learning outcomes are a step to overcome disparities in achieving the intended/desired objectives of educational planning. The Stakeholders may adopt/adapt such materials as per their needs and contexts. These can be used to lay down curricular expectations, particularly in geometry at secondary stage to provide insights into the progression of child's learning. It may be useful to teachers, parents and the entire system for learning geometry at secondary stage.

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