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Searching for Didactical Negotiations in Mathematics Textbooks

Abstract

This article offers guidelines for an objective, systematic review of mathematics textbooks for their didactical dispositions. Based on the recommendations of the Position Paper of the National Focus Group on Teaching of Mathematics brought out by the NCERT 2006, two categories for such an analysis have been proposed. In the first category the pedagogical aspects specific for building the disciplinary understanding of mathematics have been enumerated, while the second category suggest analyzing the approaches that textbooks adopt for handling the socio-psychological concerns related to the child.

Introduction

Undoubtedly, textbooks are and will remain to be the most influential instruments of education, in schools and otherwise. Textbooks reflect a Nation's vision and foundation on its education and so to some extent, they could be called as official sources of knowledge. A textbook inspires students and teachers to pursue the subject ahead. In many ways, the text gives the foundational perspective towards the nature of the subject.

In our country where teaching resources are limited, textbooks become the sole representer of the subject. In mathematics, especially, textbooks are often revered for being an esteemed source of knowledge and the expectations are centred on cultivating and strengthening a strong mathematical base, preserving the mathematical knowledge. Mathematics textbooks are deemed to provide an expertise on the subject matter, providing a sense of security to both teachers and students. For many, a mathematics textbook enjoy the status of a holy book, to be followed unquestioned.

A commonly held belief related to the nature of mathematics is of it being static, built on an edifice of knowledge accumulated over years. While the nature of the school mathematics content may be old, the processes of doing mathematics are dynamic and evolving. These two contradictory situations, of what is to be known and how to make it known, is expected to be communicated through a textbook. In other words, textbooks not only serve as sources for building students' knowledge, they also provide guidelines for teachers on what to teach and how to do so. Thus, along with the content knowledge, a textbook also contains didactical information that guides the acts of teaching and learning. The knowledge present in the textbooks have to be communicated to the user and this encompasses the didactical aspect of the textbooks. The way a textbook is written reflects on the didactical position it assumes. For

example, if the didactical position while framing a textbook is that mathematical knowledge has to be 'taught' and that it can only be learned by repeatedly working on similar problems, it is likely the textbook will contain a large number of routine questions to be practiced. Thus, the didactical aspects present in the book (may be, subtly) are significant in transmitting the meaning of the discipline. For a resource that is valued so much, it is imperative to analyse it objectively to understand how it conveys the meaning of the discipline and what didactical positions does it take to express the dispositions related to the discipline.

In this article, we propose a compilation of didactical aspects, categorised in two dimensions that may serve as a reference while framing guidelines for developing or revising upper primary mathematics textbooks. This work is aimed to serve as a guide for a systematic review of a mathematics textbook for their pedagogical inputs.

Methodological Approaches for Conducting Textbook Analysis

Textbooks analysis has been an unexplored and under-represented area, and doing one in mathematics is a farfetched idea. Commonly, the responsibility of conducting textbooks analysis is taken up by the institutes and organisations involved in textbook development such as NCERTs and SCERTs. This exercise, termed as a revision exercise, is usually done periodically by the textbook developers with an intent of re-looking at the text for any factual, numerical or procedural gaps and errors. The focus is on eliminating errors, distortions, and printing mistakes that could have gone in inadvertently. In all such exercises, one would, typically, be looking for mistakes and gaps to be rectified in future editions.

The above-stated type of textbook analysis is essential but it serves only a functional aspect as its scope is limited to determining the authenticity and appropriateness of a text. A textbook serves a rather wider and profound objective than being merely functional. Since it a medium via which the educational goals are mediated by the teachers to students, it is important that the analysis of textbooks is also done with respect to the curriculum goals. A research-based outlook will perhaps help in such a crucial review of textbooks.

While conducting textbook analysis research, one can record findings quantitatively or qualitatively. In a quantitative approach, one would enumerate the themes and measure the frequency of appearance of a particular term, phrase or text. For example, to analyse the preferred communicative style of the authors, a count of commonly occurring instructional phrases such as 'elaborate', 'prove', 'answer' may be looked at and enumerated. A quantitative analysis may also involve how much (or how little) a particular topic, idea, concept has been covered. One may say, quantitative approaches will ascertain the breadth of coverage in the textbook. Qualitative approach, on the other hand, ascertains depth over breadth. It is concerned about the way in which an information has been presented.

Varied qualitative approaches have been used to analyse mathematics textbooks. Hermeneutic analysis was used by Schubring (1987) to uncover the hidden meanings and messages in the textbooks from a historical perspective. The linguistic analysis examines the nature of the phrases, words, terminology used to communicate between the authors and the textbook users. In most of the studies based on linguistic analysis, mathematical word

problems are studied for their levels of difficulty in reading, comprehending and translating in mathematical symbolism. Gerofsky's (1996) work is one such example where linguistics structures of the word problems were studied as a genre of conceptual objects. A discourse analytic framework illuminates the relationship between the author and the reader. Herbel-Eisenmann (2007) and his colleagues (2007) uncovered how textbooks position readers and their relationship with others. Weinberg & Wiesner (2011) followed a similar approach of reader-oriented theory to express if the text assumes readers as active participants in constructing meaning while reading it. Micheal Apple propounded the critical analysis framework to expose the politics of knowledge and social inequalities in the textbooks through his various papers (some for reference are Apple, 1992; Apple & Cristian-Smith, 1991). At times, as part of this exercise, aspects related to inclusiveness, gender neutrality, biases for any particular social or political views are also checked. Thus, such an analysis could be considered as a corrective measure rather than being suggestive. One of the most commonly found research framework for analysing mathematics books is the content analysis. It is primarily concerned with representation or under-representation of a content area in the book. A content analysis of textbook unearths the intentions of the authors or book writing committee and examines which information or topics have been valued, taken for granted or dropped out considering unimportant. It is related to issues of what does the text actually cover? Does it do so sufficiently? Which topics are included and which have been omitted? And why? How have the topics been sequenced? It focuses on issues of how various mathematical content areas are handled in the textbooks, the spread of the content matter in a book or across the books (for illustration, Pickle, 2012; Reys et al. 1996; Stylianides, 2009) and comparing the depth and spread of one or more than one area across countries (see for example, Fan and Zhu, 2007; Harries and Sutherland, 1999). Along with the content, a textbook also expresses the ways of teaching mathematics and this is termed as didactical analysis. Didactics is related to the processes of teaching and is thus subject specific. It tries to answer how the textbooks convey the essence of learning the subject. This type of analysis is concerned about knowing the disciplinary perspectives, its processes and structures as delivered through the textbooks. It helps to identify exactly how the nature of the discipline has been communicated. In short, the didactic analysis of mathematics textbooks would deal with the pedagogic approaches and strategies suggestive for teaching of mathematics while the content analysis would examine the mathematical text itself.

In this paper, we share a framework that delineates the didactic concerns expressed in the Position Paper of the National Focus Group on the Teaching of Mathematics, (NCERT, 2006) and offer their representation in the mathematics textbooks.

Textbook Analysis for Didactical Concerns

Our current mathematics books are based on the recommendations stated in the Position Paper of the National Focus Group on the Teaching of Mathematics, NCERT, 2006. The paper took a deep cognisance of most of the problems that impede effective teaching and put forward a vision for learning the subject. The vision was loud and

clear "to mathematise the child's mind". A most crucial element that links this envisioned objective to implementable terms in the classrooms are the mathematics textbooks. Since the position paper has been instrumental in guiding the textbooks and bringing in a newer perspective on learning and teaching of the subject, it is imperative we look at our mathematics textbooks to analyse how effectively they have been able to interpret the intentions of the curriculum. Have the textbooks been able to meet the didactical recommendations of the curriculum?

Based on the recommendations of the position paper, the proposed analytical framework is categorised in two categories and within each category we offer guiding points that can be used for evaluation. The first category is related to the pedagogical aspects specific to the discipline of mathematics. It is concerned with the approaches that are adopted for bringing in a disciplinary understanding. Category two offers an analysis of the socio-psychological demands of the child and examines the extent to which these have been handled in the textbook.

Disciplinary Dispositions

To reach a common accord on which acts promote mathematisation in a textbook is a rather difficult task to comprehend. The expectation of the position paper that mathematics must qualitatively modify the world view of the learner requires greater immersion in the context and constant back and forth with it. The extent of relating to life and the concern about narrowing the mathematical object to the presented context challenges the effort to extend the relationship of mathematics with the life of the learner and there is no clear agreement on its form and extent as yet. However, there seems to be a consensus on what must not be encouraged in a mathematics classroom. Surely, acts of mindless memorisation, unconnected contexts, overemphasis on proofs, endorsing only one way of approaching problems and such have to be discouraged. This larger aim must also get reflected in the textbooks.

A fundamental shift has to be made from doing procedural repetitions to forming conceptual understanding. It is to bring an awareness that there can be more than one way of solving a problem and so alternative algorithms and strategies for solving a problem must be encouraged, provided they are rooted logically. Textbooks that offer such opportunities will list more than one procedure of approaching a problem and draw out the similarities among the algorithms, encourage identification of key conceptual ideas upon which the procedures have been formulated. The style or presentation would range from providing semi-closed to complete open-ended procedures. One may also find problems that have a scope for many different correct solutions.

A textbook that provides ready-made proofs will indeed close avenues for children to grapple and learn. On the other hand, textbooks following an inquiry generating mode offer spaces for building arguments, logical connections, reasoning and encouraging different ideas. Fundamental to such initiations is providing open texts for building logical arguments, without the fear of being correct or incorrect. Some of the ways for encouraging proof creation is by asking for verification of statements and readers' explanations on their verification processes. A conscious effort must be made to bring out the differences between verification and proof. Some simple, easy to understand proofs could be used to make the reader aware of what proof means.

Ample opportunities for children to observe patterns and make generalisations are a must. Along with these, spaces for identifying exceptions to generalisations and for extending the patterns to new situations encourage self-verification. The text could be written in a form that encourages children to explore the mathematical ideas, make conjectures and then move on to check the validity of the observations.

No rote learning but understanding and articulating in own words. In all aspects, it should be noted that mathematics should be presented as an emerging subject that has provisions for exploration and creation rather than following old, and often convoluted problems that hold their edifice on un-understood processes. The acts of doing mathematics also follow the style of writing the text. The communication style used in the textbook reflects on how the authors have positioned themselves and the reader in the acts of doing mathematics. Rather than following an authoritative approach, giving one suggestion to be followed by all, a discursive mode must be followed that presents alternatives and different interpretations

Children have to grapple with mathematical objects, concepts and the interrelationships among them, they must therefore engage in making meaning of definitions. For this, therefore care must be taken to not state the definitions in their purest formal form, but to let children articulate the idea in their own words. A scope of peer interaction, discussion, exchange of ideas helps children to derive invariant properties and thus form their own definitions. There would be an expectation that the children would verbalise their understanding, generalizations, and formulations of concepts to propose and improve their definitions. To do this they must be required to and encouraged to use their understandings in varied contexts including non-classroom or text-book directed ones too. This would strengthen and deepen their relationships with them. It may be pointed out that these self-articulations must arise from a conceptual framework that are aligned to and consistent with the mathematical objects as normatively understood. And for each learner the body of the constructed understanding and articulation need to be consistent within itself as well. Given the varied experiences, expectations and dispositions of the learners this is not easy to construct in a classroom and even more difficult to reflect in a textbook.

The position paper iterates that for long formal problem solving as a process of mathematical thinking has been misunderstood. It has been confined to solving textbook exercises and sadly "textbook problems reduce solutions to knowledge of specific tricks, of no validity outside the lesson where they are located" (NCERT, 2006 p.2). To make problem solving a mathematically involving exercise encourage using abstraction, quantification, analogy, case analysis, reduction to simpler situations, guess and verificational modes. These didactical aspects are important factors in promoting problem solving skills. Textbooks that follow an open writing style, providing scope for sharing of different strategies of solving problems will help achieve this goal. Text should draw analogies from earlier learnt ideas and contexts. Concomitant to problem solving is problem posing. "Mathematics also provides an opportunity to make up interesting problems, and create new dialogues thereby" (NCERT, 2006, p. 2) Textbooks rarely provide opportunities for children to form their own problems.

Creating new problems, modifying existing problems, looking for invariant and varying conditions in a problem open avenues for making one's own mathematics. A 'if-not-this, then what' is one of the efficient strategies that may be adopted in textbooks to systematically generate newer problems. Providing scope for generating problems not only inculcates a feeling of ownership but also initiates an inquiry mode in the classroom wherein children can share their posed problems and form deductions. In textbooks, opportunities to pose problems could be provided by giving open-problems, asking for identification of variant and invariant conditions in a problem and making modifications in them. Illustrating ill-structured problems for correction, providing answers and asking students to generate problem situations are some of the other ways that open windows to children's thinking.

Heuristics or rule of thumb become handy when exact answers or route to solutions is unknown. A quick glance at the textbooks would reveal if the book recognises this aspect of reaching the answers. Do the book illustrates examples where we may not look for an exact answer but seek for a closet possible answer? Are there spaces in the book where a child has to choose best possible solution among a set of possible solutions and then provide justification of the choice? A book that ensures these invites clarity of thought and an understanding when and how a mathematical technique is to be used.

"Visualisation and representation are again skills unaddressed outside mathematics curriculum, and hence mathematics needs to develop these far more consciously than is done now" (NCERT, 2006 p. 10). To meet this envisioned target, books must provide spaces for children to elucidate the underlying mathematical idea of a problem and express it in various forms such as words, pictures, symbols etc. Multiple representations of a problem induces a sense of familiarity and thus a profound understanding. Indeed, visualization aids in the processing of information and thus multiple representations of a situation in the form of pictures, images, diagrams, symbols support in thinking and developing advance understanding.

Socio-psychological Intentions

Matching the text with the social, cognitive and mathematical level of the reader is an important aspect of a good book. The entire material should be immersed in and emerge from the amalgamation of three contexts related to the child- social, cognitive and mathematical. The features that may influence this include language, nature of descriptions and examples, inclusion or lack of illustrations, visuals to illustrate a point, stories and other interesting texts and contexts. Contexts given in word situations or otherwise must be in close relation to the worlds of the children. Examples must be drawn from the child's daily activities that involve mathematical negotiations. Explanation, examples, and questions must emphasize on meaning-making, starting with experienced situations, allowing students to relate them to their respective contexts and explore their own ways of solving problems. It must be seen whether there are sufficient, meaningful and accessible spaces in the book through which a child can learn by doing, getting actively involved in the process of constructing a mathematical idea. Making connections, within mathematics, between mathematics and other subjects of study ensures relevance.

The textbook must also expect that the teachers would formulate many contextual and contextually needed

problems matching the experience and needs of the children of the class.

The emphasis in the designing of the material should be on using a language that the child can and would be expected to understand herself and would be required to work upon in a group. The teacher to only provide support and facilitation. Diversity among children suggests a greater space in the text book for the teacher and children to bring in their own particular context. The linkage that each learner has to built and her own particular nature of conceptual formulations suggests providing multiple entry points. A spiraling program to compensate for the specific hierarchical sequence and linked conceptual framework that mathematics also has much more than other disciplines. And to enable mathematisation, textbook needs to allow the teacher help children to link up to other aspects in their life and to make newer connections with it to interact with the world around more effectively.

A text-book thus needs to not only provide the space to the teacher to explore with the children and be creative in building discourses and problem sets but it must indicate to her the need to have children explore and be creative as well. The gentle nudge can be in the form of suggestions, illustrations of children engaged in a mathematics classroom to illustrate possible ways etc and it can be hard wired in to the way the problem sets are designed and the specific requirement that teachers set new problems and that the students must think of some new problems and that students are to articulate and write their understanding and even their own definition. And all this can be put in to the end of the chapter and in between exercises. The suggestive assessment criteria and examples are also important ways of nudging the process and clarifying the fundamental purpose of assessment to be of providing aid to learning.

Concluding Remarks

We hope these guidelines will prove to be helpful in the developing and revision stages of mathematics textbooks as these can be used as effective tools for initiating a rational debate on the pedagogical positions that become part of any mathematics textbook. The guidelines will serve to be functional, implementable and valuable for reference as they could be taken for a quick, but profound overview of the different didactical aspects that must be taken into account in a mathematics textbook deemed to promote mathematical thinking.

Finally, we conclude on a note that accentuates the role of an effective teacher as being crucial. No matter how elaborate a textbook maybe, it would inevitably lack in personification. Being based on a hypothetical audience that assumes the role of teachers, students, classrooms and resources, textbooks are certain to be pseudo-contextualized and pseudo-personalised (Kang & Kilpatrick, 1992). The task of facilitating a profound understanding, of making knowledge decontextualized and re-personalized would always be the responsibility of the teacher (ibid). Textbooks are written with the intention to guide teachers on the content. They give explanations and exercises as examples to be done and serve as mere guidelines and so must not be mistaken for being exhaustive. While the text book may also try to build in the image of requiring, filling in and colouring by the teachers and the children, it is up to a teacher to use the text as a springboard for promoting mathematical arguments and thoughts, offering opportunities for mathematisation. And this when the text book does not specifically seek such an input.

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