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Error Analysis in Basic Mathematical Operations

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Abstract

In India, mathematics is taught from the very beginning of schooling and is a compulsory subject up to Class X. The basics of mathematics are taught at the primary level (Classes I to V), which form the base for upper classes. But after the primary level, when students enter the upper primary level, they make many errors while solving basic mathematical problems if their concepts are not clear. Therefore, it is important to know the kind of errors, generally, committed by elementary level students. It is only then that they can be taught accordingly and chances of errors can be minimised. To find out the kind of errors, generally, committed by students, the researchers developed a mathematical achievement tool and administered it on 40 students studying in Class VIII at a government school in Bilaspur, Chhattisgarh. The researchers used purposive sampling technique to carry out the study. The collected data were analysed qualitatively. The result showed that primary level students make many errors while solving basic mathematical problems.

Background of the Study

In the beginning of schooling, the main focus is laid on learning a language, basic calculation and basic science. Language, mathematics and science are used in every aspect of our daily life. Mathematics, also known as 'the philosophy of life' (Yaratan and Kasapoğlu, 2012), is as necessary as literacy (Sumirattana, Makanong and Thipkong, 2017). Mathematics is one of the main subjects up to Class X. At the very basic level, it is taught through concrete objects, pictures and

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images. After that, abstract calculation starts. As it is a subject that requires thinking and reasoning, students learn to solve problems by employing various approaches and formula.

Knowledge of Mathematics

Mathematics helps in planning and solving problems in our day-to-day life (Phonapichat, Wongwanich and Sujiva, 2014). Therefore, it holds an important place in the school curriculum. Reading and thinking are the key factors involved in solving a mathematical problem. In order to solve a mathematical problem, the students need to acquire reading skills like reading comprehension and analysing the text, which help them to interpret the text and keywords used in a problem (Phonapichat and Wongwanich, 2014)

Literature Review

Many researches have been carried out in areas of mathematical difficulties faced by students, errors committed by them and their achievement. Suydam and Weaver (1977), and West (1977) found the difficulties that, generally, arose while solving mathematical problems. They concluded that the biggest obstacles in problem solving were due to lack of reading, computational and mathematical skills. Phonapichat, Wongwanich and Sujiva (2014) concluded that:

- students faced difficulty in understanding keywords used in problems, and were hence, unable to interpret them into a mathematical sentence.
- the students were unable to figure out what to assume and what information from the problem was necessary to solve it.
- when the students were unable to understand a problem, they solved it by guessing without applying any logical reasoning.
- the students became impatient at the time of solving mathematical problems.

Sarwadi and Shahrill (2014) found dislocation of decimal to be a major problem among students. Walker, Zhang and Surber (2008) found that reading difficulties have a significant impact on students with low mathematical achievement. In this regard, Helwing, et al. (1999) concluded reading skill to be helpful in understanding text-based mathematical problems.

Rationale of the Study

As already mentioned, mathematics is an inseparable part of our daily life. However, it must be understood by both teachers and students that mathematical problems involve much more than just placing digits for calculation. Making mathematical mistakes is common. It is a teacher's job to identify the common errors committed by students and try to eliminate or minimise their occurrence. Therefore, identification of errors, generally, committed by students

in the early years of the elementary level is important for ensuring quality learning. Attention must be paid while teaching basic mathematical concepts to students. If there is a gap in understanding the concepts, then the students may continue to face the problems throughout their lives.

This paper tries to find out some common errors committed by elementary level students while solving mathematical problems. It also aims to help teachers identify areas that many students find difficult to understand. The study has been conducted to find answers to the following research question.

Research question

What are the kind of errors, generally, committed by elementary level students while solving basic mathematical problems?

Research objectives

- To find out the errors committed by elementary level students in doing simple mathematical operations
- To find out the errors committed by elementary level students in the interpretation of mathematical problems

Methodology

Method

Qualitative research paradigm has been used to analyse the collected data.

Sample

Forty students studying in Class VIII in different government schools of Bilaspur district, Chhattisgarh, were selected for the study. Purposive sampling technique was used to conduct the study.

Tool

For the collection of data, the researchers developed a questionnaire, comprising questions requiring basic mathematical operations. This questionnaire was used for taking the mathematical achievement test of 40 Class VIII government school students in Bilaspur, Chhattisgarh.

Analysis and Interpretation of Data

Objective 1

To find out the common errors committed by elementary level students while doing basic mathematical operations

Addition

Question 1

Simple addition problem was given to the students to solve. All students, except one, solved it correctly. Figure 1 shows the error committed by the student.

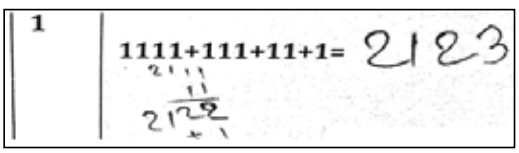

Figure 1: Error committed in solving simple addition problem

Here, the student made an error in writing the digits, placing them, and therefore, did incorrect addition. The student tried to arrange the digits. But in the first line, the student wrote 2 instead of 1, and then, did not add the second number, i.e., 111 and directly wrote 11. Therefore, due to mistake in writing in the first line, the result was wrong and it continued till the end, thereby, giving an incorrect answer. Such an error occurs if the student does not know how to place digits for calculation.

Question 2

In the next question, the students were given to solve a simple addition problem with a decimal. Maximum students (30) made errors in solving it correctly. Only six solved it correctly and the remaining four were unable to solve the problem at all. Figure 2 shows the error that most students committed while solving this question. It can be seen that the students committed an error in putting a decimal. The students did not know that the meaning of 79 is 79.00. So, they put 79 just below the second and third place after decimal, and simply did addition. Due to the wrong placement of 79, the whole process went wrong and the result was incorrect.

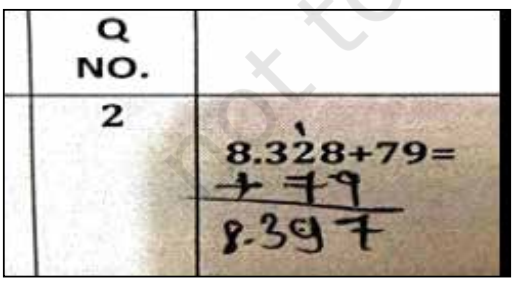

Figure 2: Error committed in solving addition problem with a decimal

Another error found in the same problem is shown in Figure 3. Here, the students added the numbers without using a decimal point. From the analysis of Questions 1 and 2, it is observed that many students made errors in solving addition problem because of wrong placement of digits with or without a decimal.

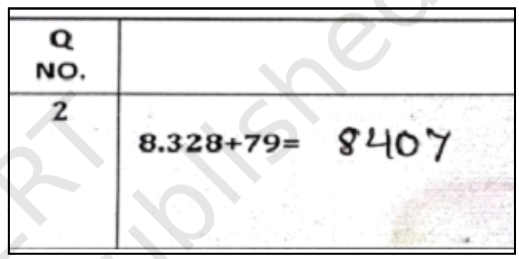

Figure 3: Error in solving addition problem with a decimal

Question 3

Students also make mistakes when the digits were arranged as in Figure 4. In this particular example, a student simply ignored the third number,

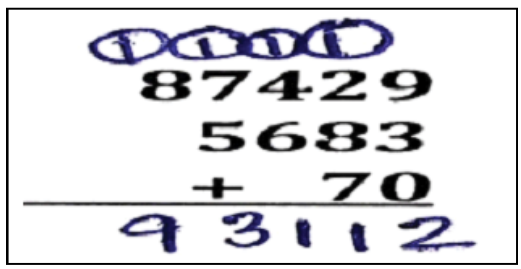

Figure 4: Error in solving addition problem when the digits are arranged correctly

i.e., 70 while calculating the value of addition of three given numbers. Similar errors were observed in the calculations done by many students. This kind of error might have occurred due to carelessness at the time of problem solving or due to limited concept of addition of three numbers having different place values.

Question 4

Here, the students were given an addition problem with a decimal. The digits were placed appropriately. Twenty-four students solved the question correctly and the remaining did it wrongly.

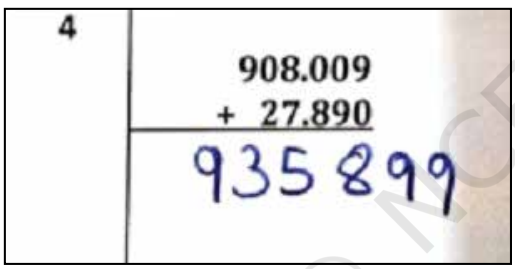

Figure 5: Error in solving addition problem with a decimal when the digits are appropriately arranged

In Figure 5, a student added the digits correctly but missed putting the decimal. It may be because of lack of concentration or carelessness. It may also be because the student lacked an understanding of the place value of decimal. This argument is strengthened with similar incorrect answers by other students like in Figure 6. Figure 6 shows error in counting, as well as, the placement of decimal.

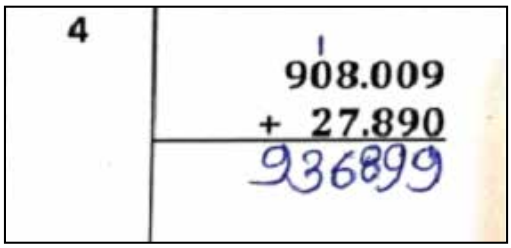

Figure 6: Error in solving addition problem with a decimal when the digits are appropriately arranged

Subtraction

Question 5

In subtraction problems, too, most students committed errors. A simple subtraction problem as shown in Question 5 was given to the students. Only 15 were able to solve it correctly, while the remaining 25 did it wrongly. The errors committed by the students in this problem are shown in Figure 7 and 8.

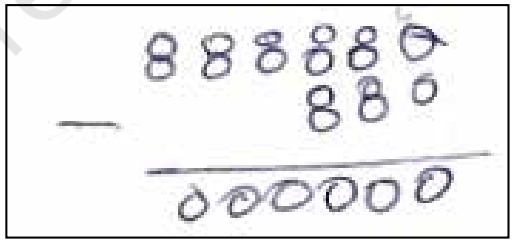

Figure 7

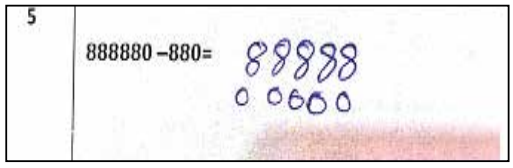

Figure 8

Figure 7 and 8: Errors in solving simple subtraction problem

Figure 7 and 8 show that the students did not know how to do subtraction. Lack of instruction may be one of the causes behind such errors.

Question 6

When they were asked to solve a subtraction problem with a decimal, the following errors were found. In Figure 9, 10 and 11, the students added the digits instead of subtracting.

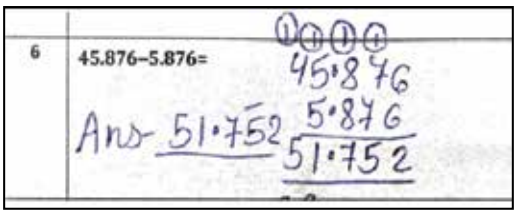

Figure 9

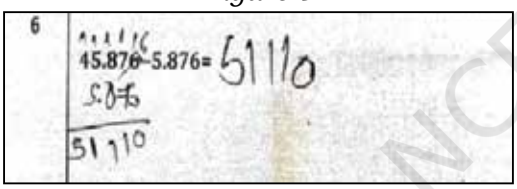

Figure 10

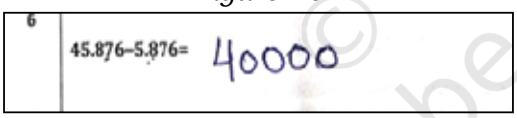

Figure 11

Figure 9, 10 and 11: Errors in solving subtraction problem with decimal

Question 7

Figure 12, 13 and 14 show some of the responses of students to a subtraction problem as regards to whole numbers. In all cases, it is errors in calculation. A deeper inspection reveals that the results were wrong due to limited understanding in 'borrowing of numbers', and of course, inability to recall steps of operation.

In Figure 12, many students did subtraction of whole numbers, i.e., numbers without decimal. But the result was wrong. Figure 14 shows that a student perhaps read 9000 as 900, which depicts problem with attention and working memory, and rounded off 99 as 100, diluting the preciseness of mathematical operation. From Figure 13 and 14, it can be concluded that the students made mistakes in calculating. However, they knew how to do subtraction.

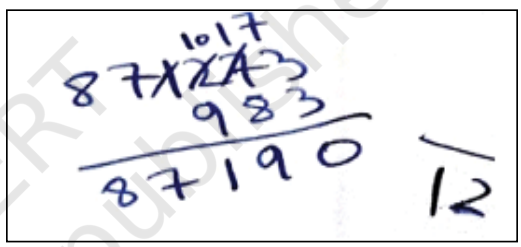

Figure 12

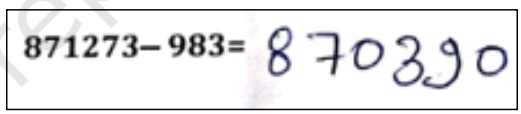

Figure 13

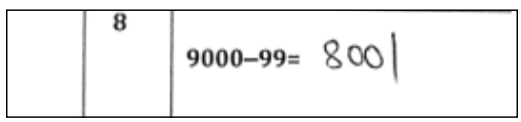

Figure 14

Figure 12, 13 and 14: Calculation error in solving subtraction problem

Question 8

In Figure 15, another student correctly borrowed numbers for subtraction of a bigger number from 0 and solved it correctly up to three places. However,

at the fourth place, the value borrowed was missed, and hence, the result was incorrect. The student did not even notice that the subtraction value was higher than the positive numbers involved in the process. Apart from carelessness, it also shows lack of sustained attention to allow the working memory to follow correct steps of action for certain duration.

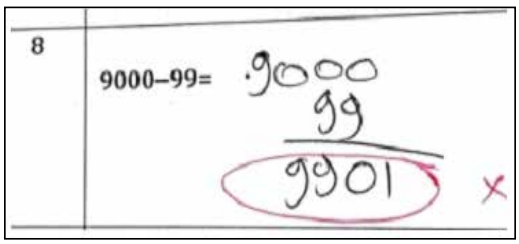

Figure 15: Calculation error in solving subtraction problem without decimal

Multiplication

Question 9

In multiplication, maximum students did not know the rules of multiplying. A total of 29 out of 40 students wrote the same digits (Figure 16) only instead of zero (0). This may be due to lack of clear instruction.

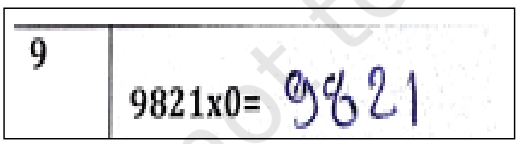

Figure 16: Error in solving multiplication problem

Question 10

Many students committed errors in multiplying digits with a decimal.

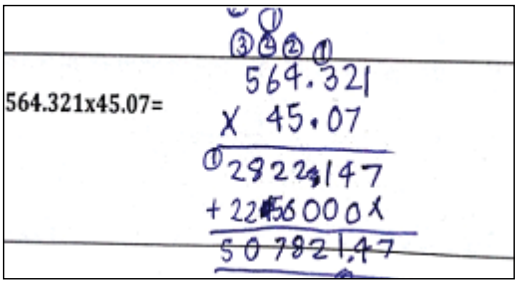

Figure 17: Error in solving multiplication problem with decimal

In Figure 17, many students committed an error in the table of 7. This can be seen in the first line itself. Here, the answer of one student indicates that the child started correctly for ones and tens but did not carry over the remaining for the digit placed in hundreds and wrote 21 (i.e., 7 muliplied by 3), and then, the table of 7 is not correctly done for the remaining digits.

In the second line, the student made a mistake in multiplication with zero. Here, the student, started correctly from the right side but after multiplying the three digits, error was made in multiplying the remaining digits with 0. Further, the student also did not complete the multiplication. The child, then, added them and gave the final result of the problem. It is also clear that the student had no idea of decimal placement. When the student was asked to multiply, the child responded of not being able to solve multiplication problems correctly due to mathematics phobia. The child admitted to running away from the subject. Such kind of result may be due to difficulty or inability in recalling

or remembering and practising. Hence, it can be concluded that the students need to memorise the tables first so that they are able to solve multiplication problems correctly.

Question 11

Figure 18(a) shows another example of multiplication. Here, one student multiplied the digits on the right side of decimal but wrote the left side digits in the first line.

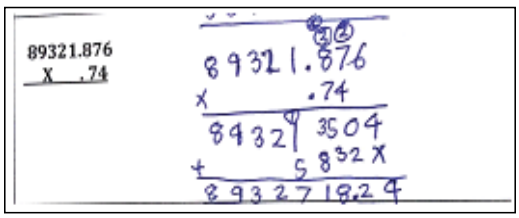

Figure 18(a): Error in solving multiplication problem

In the next section, too, one student multiplied correctly (digits from the right side of the decimal) but wrote the same digits (left side of the decimal). The student, then, found the solution by adding the above two lines. At last, the student put the decimal in the wrong place. This is because of lack of knowledge and understanding of the concept of multiplication.

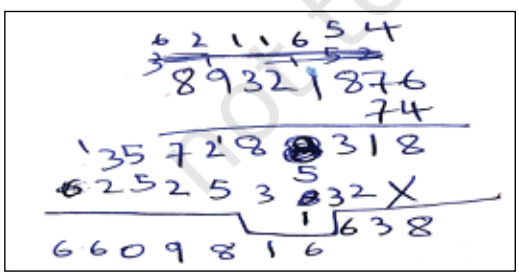

Figure 18(b): Error in solving multiplication problem

Figure 18(b) also shows error in the same problem of multiplication by another student. In the first line of the solution, it is evident that the student does not know the table of 4. This can be seen in the starting (multiplication of 6 with 4). As a result, the further multiplication went wrong. Same kind of error is made in the next line as well. Such errors occur due to inability to memorise and remember tables.

Division

Question 12

In Figure 19 and 20, the students made a mistake in subtracting in a division problem, which is seen in the first line itself. It is also found that two

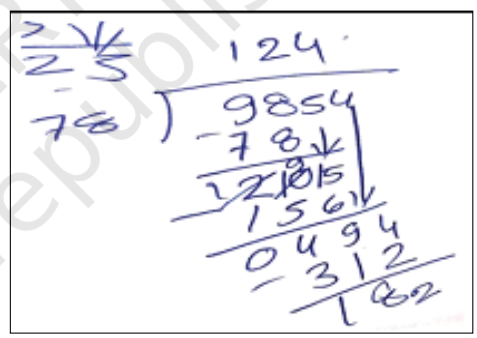

Figure 19

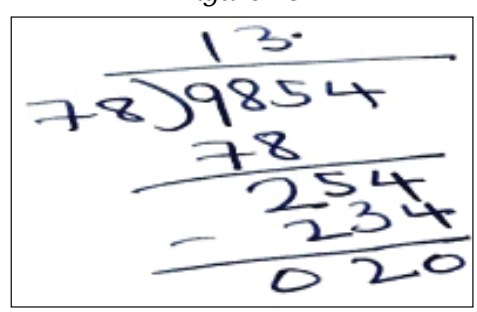

Figure 20

Figure 19 and 20: Errors done in solving division problem

students were unable to find out the divisible number, which is proven from the last line of the solution as shown in Figure 19.

In Figure 21, two students did an error in subtraction, which is an important step in division. This can be seen in the first line of the solution, i.e., 78 is subtracted from 98, and instead of writing 20, only 2 is written and the next number is brought down from above and a 0 is put next to it. Another error found here is in the placement of decimal. The decimal is placed in the wrong place. This example also shows lack of understanding. While solving a division problem, a zero comes only when the remainder is indivisible and only after that a decimal is required in the quotient. We can say that the student does not understand the concept of division.

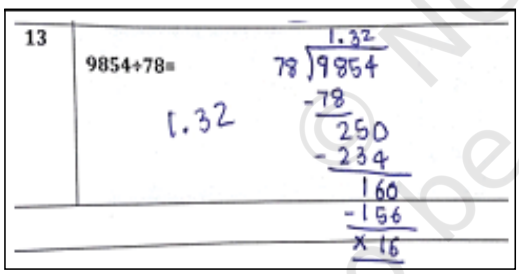

Figure 21 Third example of error in solving division problem

Question 13

In Figure 22, 28 students committed an error in subtraction. It is a frequent error made by students at the time of solving a division problem. This may be because the concept of division is not clear and also due to carelessness at the time of problem solving.

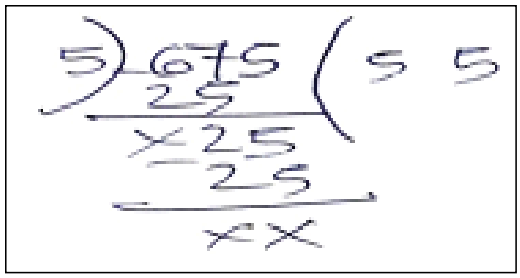

Figure 22: Error in solving division problem

Objective 2

In case of word problems, errors are found not only in the process of solving the problems but also in the statement of reporting the solution or result.

In the first question, it was given that Anamika has 57 mangoes. Her sister Alisha gives her some more mangoes. Now, Anamika has a total 108 mangoes. Find out how many mangoes did Alisha give Anamika?

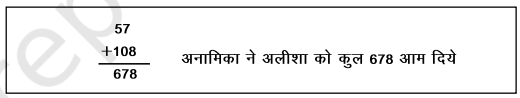

Figure 23

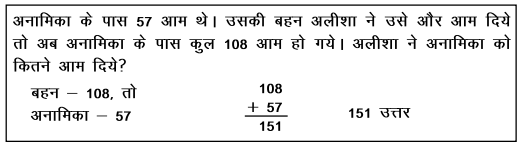

Figure 24

Figure 23 and 24: Errors in solving addition (word) problem

Fifteen students attempted to answer the word problem on subtraction. But as shown in Figure 23 and 24, the students did addition instead of subtraction. Also, one can note from both the cases, the written statement of the problem as proposed

by the student (Figure 24) or report of the solution (Figure 23), the meaning conveyed goes opposite to that of the problem. The evidences indicate that errors in mathematics are not only due to lack of understanding of the calculation process but also due to limited ability to interpret the statement of word problems.

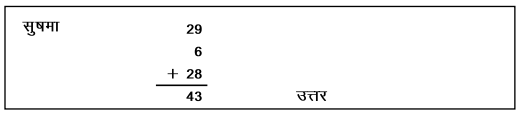

Figure 25

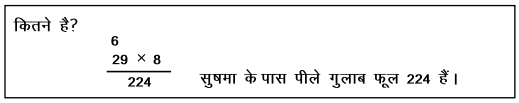

Figure 26

Figure 25 and 26: Errors committed by students while solving word problems of subtraction

In the second question, it was stated that Sushma has 29 flowers. Of these, six are red, eight are pink and the rest are yellow. How many yellow flowers does she have?

Twenty-three students added up all the flowers. Their addition was correct but it was not applicable here (Figure 25). Two students did multiplication as shown in Figure 26.

Eleven students used the correct process but they made a mistake in calculation. Only four students solved the problem correctly. The students, who made mistakes, did not read the problem carefully, and hence, were unable to understand and solve it correctly.

In the third question, it was asked that Sujata has 105 sweets. She has to distribute it among three sections of Class IX. Figure 27, 28, 29, 30 and 31 show the errors made in solving word problems of division by students.

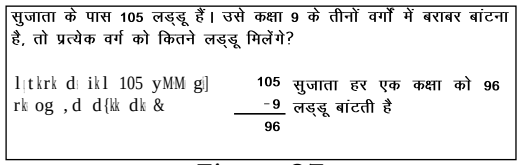

Figure 27

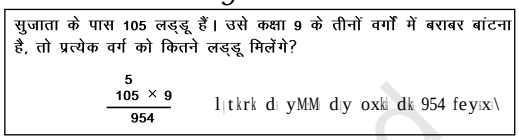

Figure 28

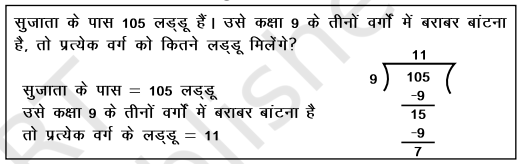

Figure 29

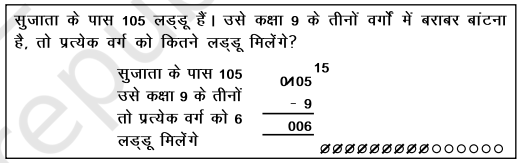

Figure 30

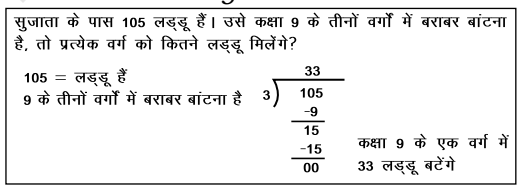

Figure 31

Figure 27, 28, 29, 30 and 31: Errors committed by students in solving word problems of division

Five students solved the problem by subtracting 9 from 105 (Figure 27), 11 multiplied 105 and 9 (Figure 28) and 18 divided 105 by 9 (Figure 29). One student subtracted 9 from 105

(Figure 30) and the remaining five divided 105 by 3, but there were errors in the division (Figure 31).

From the analysis of the responses of the students and the corresponding figures, it can be said that they were unable to understand mathematical problems in statement form.

In the last question, they were asked that Ramesh has 46 packets of chocolates and each packet contains 14 chocolates. How many chocolates does Ramesh have? In this question, instead of multiplying 46 with 14, 25 students did addition, four divided 46 by 14 and two subtracted 46 from 14. Figure 32 and 33 show the errors made in solving the problem.

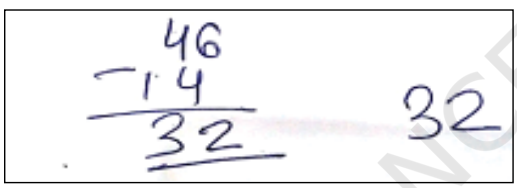

Figure 32

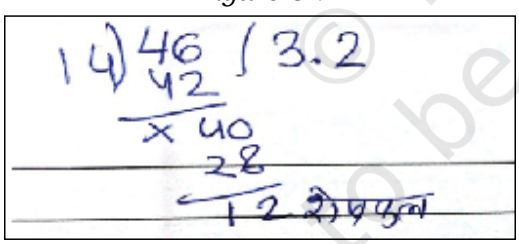

Figure 33

Figure 32 and 33: Errors in solving word problem of multiplication

Discussion

It is found that students commit many errors in solving basic mathematical problems. Some of the common errors they make are as follows.

Unable to calculate

As many students are unable to calculate correctly, there is a need to teach them methods to do calculations.

Errors in the placement of decimal

It has been observed that many students are confused in the placement of decimal. Therefore, there is a need to teach them as regards to the placement and use of decimals in mathematics.

Unaware of subtraction rules

Many students do not know the rules and process of subtraction. Therefore, they must be taught the way to subtract.

Unable to memorise and recall tables

In multiplication and division problems, it has been found that students make mistakes at the time of recalling tables, which is the base for solving such mathematical problems. Therefore, students must be made to memorise the tables in the primary stage itself.

Poor comprehension abilities

Most students fail to understand what is given in the statement of a problem and what has to be found. This is one of the main reasons why most of them failed to solve word problems. None of the students could appropriately describe the problem in their own language. Besides, they could not report their findings in line with the problem posed in the question. However, all attempted the question, without understanding the direction

to attempt the problem and tried to do some 'calculation'. They only calculated the digits. This may be due to lack of clear instruction, practices and guidance by teachers and parents.

Implications and Conclusion

The above observations lead us to certain conclusions regarding the teaching–learning of an abstract subject like mathematics at the elementary level. Fundamental knowledge of mathematics is important to lead a productive independent life. Along with verbal language, numbers, shapes, etc., also form part of symbolic instruments to represent the world around us. Numeracy is one of the compulsory skills required even as under the 3Rs (Reading, Writing and Arithmatic). It is one of those areas, which has been part of the core curriculum and cross-curriculum in many national school curricula like in the UK and Australia. Compulsory mathematics up to the secondary level also bears the same intention in India. Lack of proficiency in skills like basic operations, place value of digits in carrying out mathematical operations, etc., deter the goal of education.

Mathematics is not merely understanding of numeracy. It also involves training of thoughts, and use and understanding of language, apart from numbers. It requires language to frame a concept. Hence, training of mathematics must involve simultaneous training of numeracy, language and thought. It is all the more significant at the elementary level of schooling as children in that age group, generally, depend on concrete operational thoughts. Confusion among students in the processes of four basic operations — addition, subtraction, multiplication and division — indicate a problem in understanding these basic mathematical concepts.

The study also highlights that traditionally adopted methods of reception learning, copying the problems solved by teachers on the blackboard, lack of continuous practice and assessment with feedback, etc., lead students to disadvantage, which eventually causes fear, disinterest and despair in them.

Teaching–learning of mathematics should not be limited to completing the exercises. Rather it should focus on training of supporting skills of thinking — starting from the basic process of paying and sustaining attention to the more complex ones like concept attainment. This training should start at the basic level. Even in the primary classes, it is necessary to adopt strategies, which give learners a scope to retain attention until they solve the problem. An interactive and enriched environment would be beneficial for the students for rule and mastery learning, and shape their concepts in a broader generalised fashion from the very beginning.

Attention should also be paid to inculcating mathematical values of preciseness and accuracy in the learners so that they learn to identify their mistakes and follow precise methods to solve mathematical problems.

Communication is a strong tool for concept building. The students need to be encouraged to think aloud how they interpret a given problem and how they would solve it. Small group discussions, peer collaborations and appropriate probing strategies can help the students enhance their reading and listening skills, as well as, give them space for self-talk to internalise the knowledge that they have reconstructed.

Lastly, mathematics curriculum goals should not be explicit only about the contents but also state the thinking skills that need to be achieved. It is high time that our school curriculum is visualised in a shape other than just a prescribed list of contents, keeping other necessary dimensions implicit. This will be helpful in ensuring holistic teaching–learning in early years of school education. As knowledge of mathematics is cumulative in nature, only a thoughtful teaching–learning in the early years of school can help the students meaningfully survive in the increasingly complex world of numbers, shape, size and estimation.

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