Physico-mathematical Conceptual Difficulties

A Barrier to Learning 'Motion' in Physics among Higher Secondary Students in Kerala

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Abstract

Physics is a difficult school discipline due to the difficulty faced by the students in integrating the concepts of mathematics and physics (Mwangala and Shumba, 2016). This study provides insights into higher secondary students' difficulties when solving physics problems involving the application of mathematical concepts. The data required for the investigation was gathered using 'Physico-mathematical Conceptual Test' with 70 multiple choice questions based on select basic concepts from 'Motion'. The test was conducted on 880 higher secondary students who were sampled using stratified sampling technique from various districts of Kerala. The study showed that the extent of physico-mathematical conceptual difficulties in total and in the sub-categories, viz., 'creating or identifying the formula', 'extracting information from diagrams or graphs', 'creating schematic diagrams or graphs' and 'application of mathematics', are moderate. The extent of each category of difficulty in the respective topics was also studied to rank them.

Introduction

"The dominating feature of the contemporary world is the intense cultivation of Science on a large scale, and its application to meet the country's requirements" (Ministry of Science and Technology, Government of India, 1958). The production of better citizens who have imbibed the value of the spirit of scientific inquiry

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to opt better standards in living is not possible without science education.

In India, through the efforts of National Council of Educational Research and Training (NCERT), Science has been made a compulsory subject throughout the school stage. The Secondary Education Commission (1952–53) had also recommended that every secondary school pupil should study general science as a compulsory subject, so that the learner gains basic quantum of scientific knowledge as a part of his general education (Ministry of Education, Government of India, 1953).

Science had been arbitrarily split into different 'streams' when the amount of information had started to increase beyond the limits where it could be considered as a single subject. But education being a comprehensive and coordinated process have to be correlated to various streams and transacted to attain the aim of 'unification of knowledge'.

There is the need of deliberate effort to be laden to integrate various disciplines and to teach them as a synthetic whole. The shift of dissemination of unified, integrated and meaningful knowledge to the limelight was the result of the child's psychological build-up which wants to receive learning experiences in an integrated manner. An example would be that of mathematics and physics. In reality, one of the most extensive applications of mathematics is in physics (Embeywa 1985; Redish, 2005). The bond between these two streams of science is a very strong one (Hutchings, 1973). In fact, some topics in physics appear exactly the way they are in mathematics (Munene, 2014). Mathematics is a principal tool of physics (Greene, 1969). So, it is clear that learning physics requires mathematical knowledge (Ataide and Greca, 2013; Bing, 2008; Redish, 2005; Vinitsky-Pinsky and Galili, 2014).

Physics is considered to be a particularly difficult school discipline due to the difficulty faced by the students in integrating the concepts of mathematics and physics (Mwangala and Shumba, 2016; Pietrocola, 2008; Tuminaro, 2004; Vinitsky-Pinsky and Galili, 2014;). This difficulty can also be observed among students in doing an appropriate calculation and also in interpreting the results of a physics problem (Tuminaro and Redish, 2007).

So, it is important to investigate the extent of difficulties of students to identify, combine and apply physics and mathematical concepts, here after, known as 'Physico-mathematical Conceptual Difficulties (PMCDs)', to make the transaction of the content and reproduction of the comprehended knowledge effectively. PMCD occur either in one or more categories, viz., 'creating or identifying the formula', 'extracting information from diagrams or graphs', 'creating schematic diagrams or graphs' and 'application of mathematics'. This study provides insights into higher secondary students' PMCDs in handling concepts under the topic 'Motion' in physics.

Research Questions

The following are the research questions of the study.

1. What is the extent of Conceptual Difficulty (CD) in 'Motion' among higher secondary students?
2. What is the relative position of Physico-mathematical Concepts based on the extent of Conceptual Difficulty (CD) among higher secondary students?
3. What is the extent of Physico-mathematical Conceptual Difficulties (PMCDs) among higher secondary students?
4. What is the relative position of concepts from 'motion' based on the extent of Physico-mathematical Conceptual Difficulties (PMCDs) among higher secondary students?
5. What is the extent of Physico-mathematical Conceptual Difficulty (PMCD) among higher secondary students in
   - (a) creating or identifying the formula?
   - (b) extracting information from diagrams or graphs?
   - (c) creating schematic diagrams or graphs?
   - (d) application of mathematics?
6. What is the relative position of concepts from 'Motion' based on the extent of Physico-mathematical Conceptual Difficulty (PMCD) among higher secondary students in—
   - (a) creating or identifying the formula?
   - (b) extracting information from diagrams or graphs?
   - (c) creating schematic diagrams or graphs?
   - (d) application of mathematics?

Methodology

Method and Sample

Survey method was conducted on a sample of 880 higher secondary school students from Kerala who have opted science as their main stream (Male: 420, Female: 460; Rural: 597, Urban: 283; Government: 482; Aided: 233, Unaided: 165) using stratified sampling technique.

Tool

The tool used for the study is 'Physico-mathematical Conceptual Test'. The test consists of 78 multiple choice questions based on the basic concepts from 'Motion' at higher secondary level viz., 'Distance', 'Displacement', 'Speed', 'Velocity', 'Acceleration', 'II Equation of Motion', 'III Equation of Motion', 'Newton's Second Law of Motion' and 'Law of Conservation of Momentum'. The test contains two items from each concept for measuring the extent of CD (i.e., total 18 items to measure CD) and each category of PMCD viz., 'Creating or Identifying the Formula', 'Extracting Information from Diagrams or Graphs', 'Creating Schematic Diagrams or Graphs' and 'Application of Mathematics' (i.e., total 60 items to measure PMCD).

A brief description of CD and the four categories of PMCDs based on which the test was prepared is given below.

Concept

This category refers to the difficulty in identifying, retrieving, recalling, recognising and selecting the correct Physico-mathematical Concepts from the given cues. It also includes the difficulty in differentiating or distinguishing one concept from another.

Example: Total length of the path travelled by a moving body is—

(i) Distance
(ii) Velocity
(iii) Displacement
(iv) Speed

Creating or Identifying the Formula

This category refers to the difficulty in creating, identifying, generating, modifying, rearranging and relating the appropriate formula and equations based on Physico-mathematical reasoning that would best represent the situation provided.

Example: Orbit of an artificial satellite at distance 42260 km from earth is circular. It completes one revolution around the earth in 24 hrs. Its speed will be—

(i) $\left[\frac{3.14 \times 42260}{2 \times 24}\right] m/s$

(ii) $\left[\frac{2 \times 3.14 \times 42260}{24}\right] m/s$

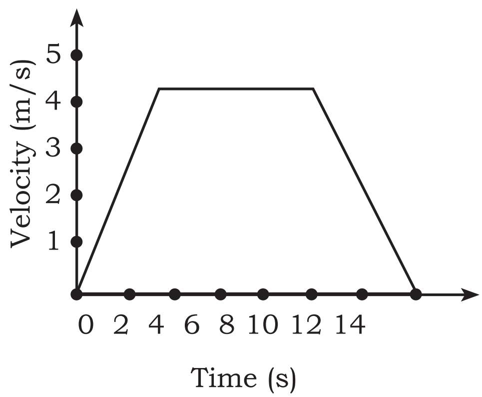
(iii) $\left[\frac{3.14 \times 42260}{2 \times 24}\right] km/hr$

(iv) $\left[\frac{2 \times 3.14 \times 42260}{24}\right] km/hr$

Extracting Information from Diagrams or Graphs

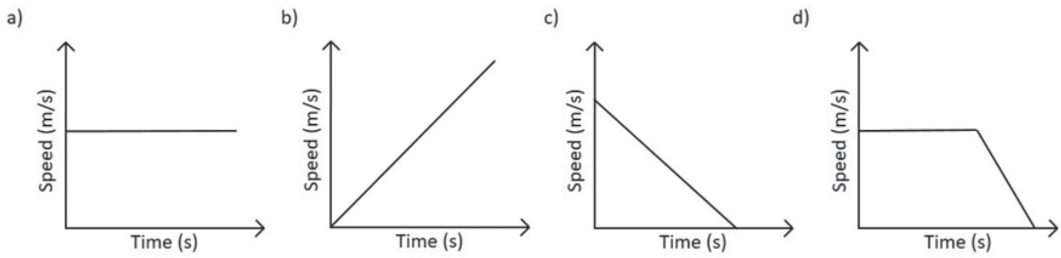
This category refers to the difficulty in decoding and extracting the correct Physico-mathematical information from the pictorial representations like schematic diagrams and graphs.

Example: Study the given graph and calculate the distance travelled by the body in first 8 seconds of the journey graphically.



Creating Schematic Diagrams or Graphs

This category implies the reverse to that of the previous difficulty discussed. This refers to the difficulty in creating; generating, designing and reconstructing the appropriate schematic diagrams or graphs that



would best represent and explain the given physico-mathematical concept.

Example: Which of the following graphs show increasing speed with time?

Application of Mathematics

This category refers to the difficulty in applying and understanding operations in mathematics so as to produce the result and solve the physics problems completely without leaving the calculations half-way.

Example: A car starting from rest moves with a uniform acceleration of 2m/for 5 min. The final velocity of the car will be—

(i) 2.5m/s	(ii) 10m/s
(iii) 7m/s	(iv) 600m/s

One mark is assigned for every correct response and zero for every incorrect response. The maximum mark obtainable on the test is 78.

The reliability of the test was ensured using split half method (r = 0.81). Face validity, content validity and criterion related validity (r = 0.65) of the test were also established.

Statistical Technique

Percentage analysis of responses of students obtained through the test was used for satisfying the objectives of the study.

Data Analysis and Interpretation

Data Preparation for Analysis

As the tool 'Physico-mathematical Conceptual Test' was prepared by setting two items in each concept under CD and the remaining four categories of PMCDs, viz., creating or identifying the formula, extracting information from diagrams or graphs, creating schematic diagrams or graphs and application of mathematics, the investigator has taken the mean of the obtained scores in the two items as the 'Average Score' for the respective difficulty.

Extent of CD

The 'Average Score' obtained in conceptual items is subtracted from the maximum obtainable average score to get the 'Index of Conceptual Difficulty' in that corresponding concept. The Indices of CD in all the concepts are summed up to obtain the 'Total Index of CD in Motion'. It is then converted to percentage to get 'Percentage Score of CD in Motion'.

Extent of PMCDs

The 'Average Score' obtained in each concept under various categories of PMCDs is subtracted from the 'Average Score' obtainable on the corresponding conceptual item to get the 'Index of PMCDs' under each category of difficulty in that concept. Only the 'Index of PMCDs' of the students who have the average score of conceptual understanding greater or equal to the average score in Physico-mathematical conceptual understanding is considered for further analysis. In each category of difficulties, the sample may vary. The 'Indices of PMCDs' of all the concepts are summed up to obtain the 'Total Index of PMCDs'. It is then converted to percentage to get 'Percentage Score of PMCDs'.

Extent of CD in Motion

The cumulative percentage frequency curve of CD in physics is given as Figure 1.

Figure 1 implies that among the higher secondary students, half of them are having 50 per cent or more CD in motion. This explains that half of the higher secondary students possess a low level of conceptual understanding in motion.

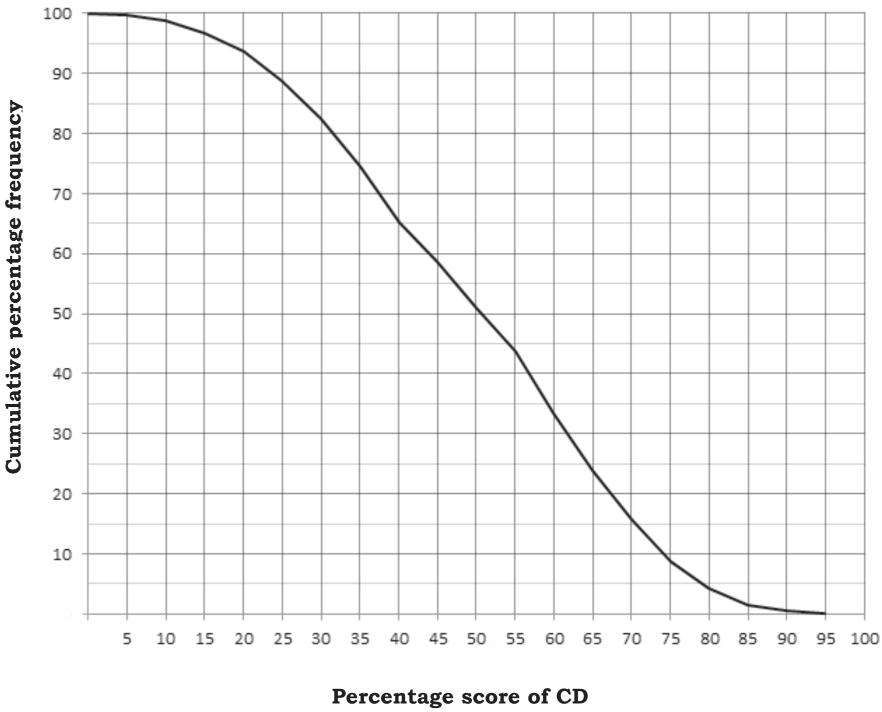


Fig. 1: Cumulative percentage frequency curve of CD in physics

Table 1: Mean Percentage Scores of CD in each Select Physico-mathematical Concepts

S. No.	Physico-mathematical concepts	Mean percentage score of CD
1.	III Equation of Motion	73.86
2.	II Equation of Motion	54.20
3.	Law of Conservation of Momentum	53.64
4.	Velocity	52.39
5.	Speed	48.75
6.	Newton's Second Law of Motion	48.01
7.	Distance	42.78
8.	Acceleration	41.48
9.	Displacement	27.95

Ranking of Concepts based on the Extent of CD

The concepts were ranked and listed based on their extent of CD among higher secondary students in Table 1. For analysing the extent of CD in each concept, the investigator has set the following criteria for interpretation. The CD is said to be high if the percentage score is greater than 50; it is moderate if the percentage score lies between 30 and 50; it is low when the percentage score is less than 30.

From Table 1, it is clear that even though the students are from the science stream, they possess high level of CD in the topics 'III Equation of Motion', 'II Equation of Motion', 'Law of Conservation of Momentum' and 'Velocity'; moderate level of CD in the topics 'Speed', 'Newton's Second Law of Motion', 'Distance' and 'Acceleration'; low level of CD in the topic 'Displacement'.

Extent of PMCDs

For interpretation of extent of PMCDs, the criteria set by the investigators is that the PMCDs is said to be high if the percentage score is greater than 30; it is moderate if the percentage score lies between 10 and 30; it is low when the percentage score is less than 10.

The cumulative percentage frequency curve of PMCDs is given as Figure 2. Figure 2 reveals that half of the higher secondary science students are having nearly 25 per cent or more PMCDs. In other words, even though the students are having conceptual understanding, half of them possess only a moderate level of Physico-mathematical Conceptual Understanding.

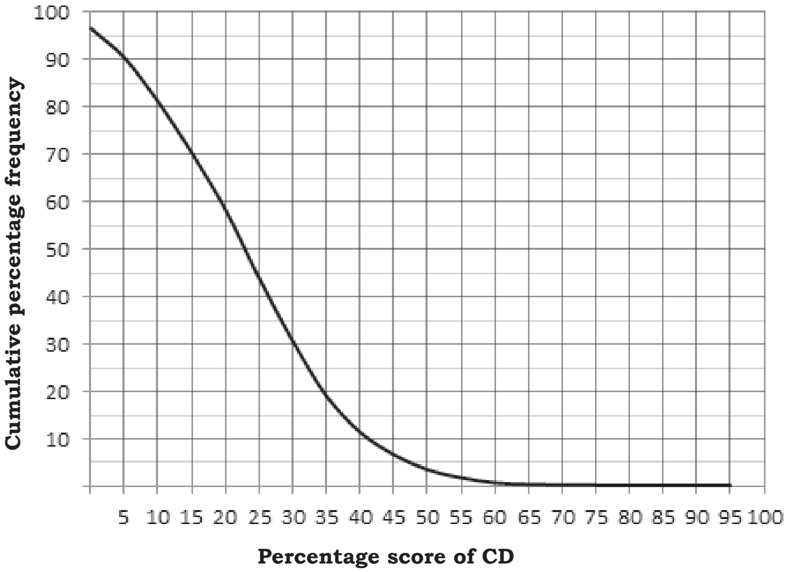


Fig. 2: Cumulative percentage frequency curve of PMCDs

Ranking of Concepts based on the Extent of PMCDs

The concepts were ranked and listed based on their extent of PMCDs among higher secondary students in Table 2.

From Table 2, it is clear that the students possess high level of PMCDs in the topics 'Displacement' and 'Acceleration'; moderate level of PMCDs in the topics 'II Equation of Motion', 'Velocity', 'Distance', 'Newton's Second Law of Motion', 'Law of Conservation of Momentum' and 'Speed'; low level of PMCDs in the topic 'III Equation of Motion'.

Table 2: Mean Percentage Scores of PMCDs in each Concepts

S. No.	Physico-mathematical concepts	Mean percentage score of PMCDs
1.	Displacement	35.34
2.	Acceleration	31.77
3.	II Equation of Motion	29.23
4.	Velocity	25.51
5.	Distance	23.35
6.	Newton's Second Law of Motion	22.78
7.	Law of Conservation of Momentum	22.22
8.	Speed	15.37
9.	III Equation of Motion	2.63

Extent of various Categories of PMCDs

Creating or Identifying the Formula

Extent of Difficulty in Creating or Identifying the Formula

The cumulative percentage frequency curve of difficulty in creating or identifying the formula is given as Figure 3.

Figure 3 implies that half of the students are having 25 per cent or more difficulty in creating or identifying the formula. This shows that even though the students are having conceptual understanding, half of them possess only a moderate level of understanding in creating or identifying the formula.

Ranking of Concepts based on the Extent of Difficulty in Creating or Identifying the Formula

The concepts were ranked and listed based on their extent of difficulty in creating or identifying the formula in Table 3.

From Table 3, it is clear that the students possess high level of difficulty in creating or identifying the formula in the topics 'Displacement', 'Newton's Second Law of Motion'

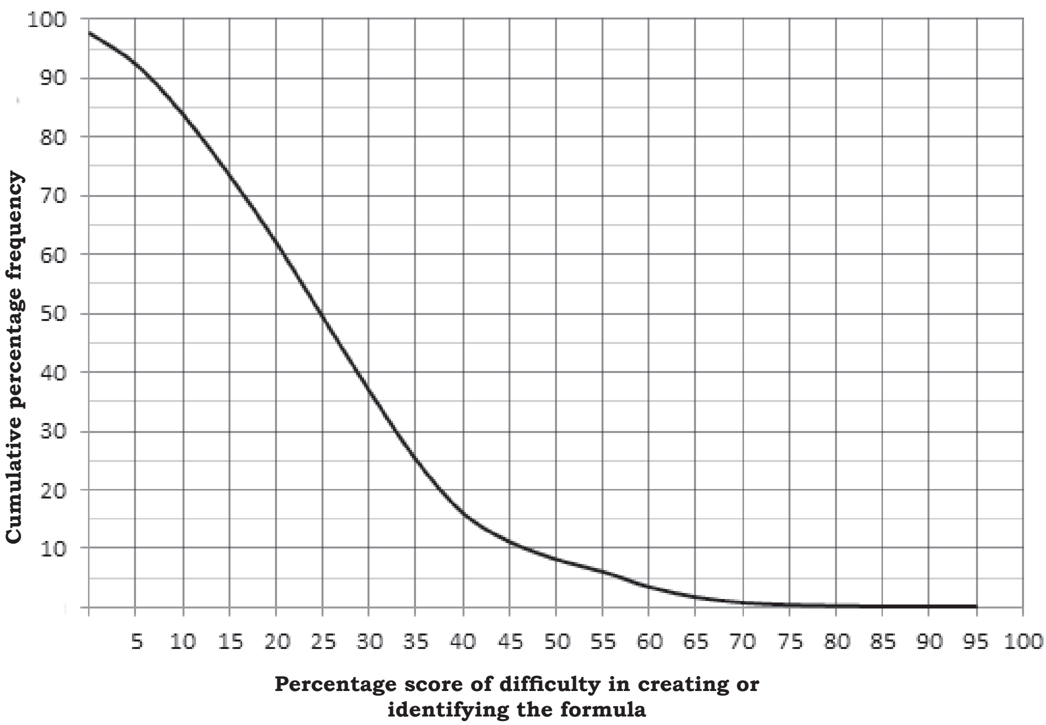


Fig. 3: Cumulative percentage frequency curve of difficulty in creating or identifying the formula

Table 3: Mean Percentage Scores of Difficulty in Creating or Identifying the Formula in Each Concepts

S. No.	Physico-mathematical Concepts	Mean percentage score of difficulty in creating or identifying the formula
1.	Displacement	44.30
2.	Newton's Second Law of Motion	36.40
3.	Velocity	34.09
4.	Acceleration	27.32
5.	Distance	27.19
6.	II Equation of Motion	22.99
7.	Law of Conservation of Momentum	18.86
8.	Speed	13.41
9.	III Equation of Motion	4.01

and 'Velocity'; moderate level of difficulty in creating or identifying the formula in the topics 'Acceleration', 'Distance', 'II Equation of Motion', 'Law of Conservation of Momentum' and 'Speed'; low level of difficulty in creating or identifying the formula in the topic 'III Equation of Motion'.

Extracting Information from Diagrams or Graphs

Extent of Difficulty in Extracting Information from Diagrams or Graphs. The cumulative percentage frequency curve of difficulty in extracting information from diagrams or graphs is given as Figure 4.

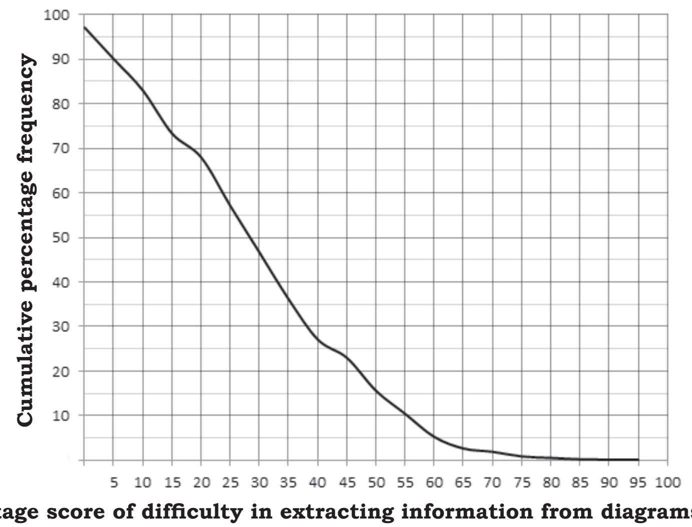


Fig. 4: Cumulative percentage frequency curve of difficulty in extracting information from diagrams or graphs

Figure 4 shows that half of the students are having 28 per cent or more difficulty in extracting information from diagrams or graphs. This implies that even though the students are having conceptual understanding, half of them possess only a moderate level of understanding in extracting information from diagrams or graphs.

Ranking of Concepts based on the Extent of Difficulty in Extracting Information from Diagrams or Graphs

The concepts were ranked and listed based on their extent of difficulty in extracting information from diagrams or graphs in Table 4.

From Table 4, it is clear that the students possess high level of difficulty in extracting information from diagrams or graphs in the topics 'Acceleration' and 'Displacement' and moderate level of difficulty in extracting information from diagrams or graphs in the topics, 'Distance', 'Speed', 'Newton's Second Law of Motion' and 'Velocity'.

Creating Schematic Diagrams or Graphs

Extent of Difficulty in Creating Schematic Diagrams or Graphs

The cumulative percentage frequency curve of difficulty in creating schematic diagrams or graphs is given as Figure 5.

Figure 5 shows that half of the students are having 22 per cent or more difficulty in creating schematic diagrams or graphs. This explains that even though the students are having conceptual understanding, half of them possess only a moderate level of understanding in creating schematic diagrams or graphs.

Ranking of Concepts based on the Extent of Difficulty in Creating Schematic Diagrams or Graphs

The select concepts were ranked and listed based on their extent of difficulty in creating schematic diagrams or graphs in Table 5.

Table 4: Mean Percentage Scores of Difficulty in Extracting Information from Diagrams or Graphs in Each Concepts

S. No.	Physico-mathematical Concepts	Mean percentage score of difficulty in extracting information from diagrams or graphs
1.	Acceleration	35.96
2.	Displacement	35.59
3.	Distance	29.32
4.	Speed	26.27
5.	Newton's Second Law of Motion	23.66
6.	Velocity	20.62

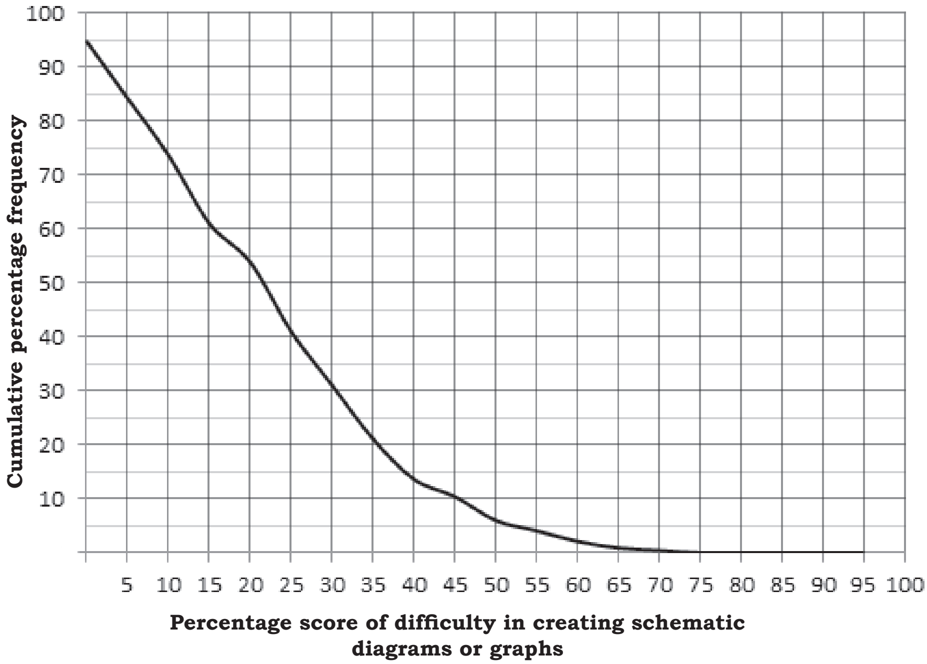


Fig. 5: Cumulative percentage frequency curve of difficulty in creating schematic diagrams or graphs

Table 5: Mean Percentage Scores of Difficulty in Creating Schematic Diagrams or Graphs in Each Concepts

S. No.	Physico-mathematical concepts	Mean percentage score of difficulty in creating schematic diagrams or graphs
1.	Velocity	29.73
2.	Displacement	29.66
3.	Newton’s Second Law of Motion	26.81
4.	Acceleration	25.89
5.	Distance	20.48
6.	Speed	3.13

From Table 5, it is clear that the students possess high level of difficulty in creating schematic diagrams or graphs in the topics ‘Velocity’ and ‘Displacement’; moderate level of difficulty in creating schematic diagrams or graphs in the topics ‘Newton’s Second Law of

Motion', 'Acceleration' and 'Distance'; low level of difficulty in creating schematic diagrams or graphs in the topic 'Speed'.

Application of Mathematics

Extent of Difficulty in Application of Mathematics

The cumulative percentage frequency curve of difficulty in application of mathematics is given as Figure 6. Figure 6 reveals that half of the students are having 24 per cent or more difficulty in application of mathematics. This shows that even though the students are having conceptual understanding, half of them possess only a moderate level of understanding in application of mathematics.

Ranking of Concepts based on the Extent of Difficulty in Application of Mathematics

The concepts were ranked and listed based on their extent of difficulty in application of mathematics in Table 6.

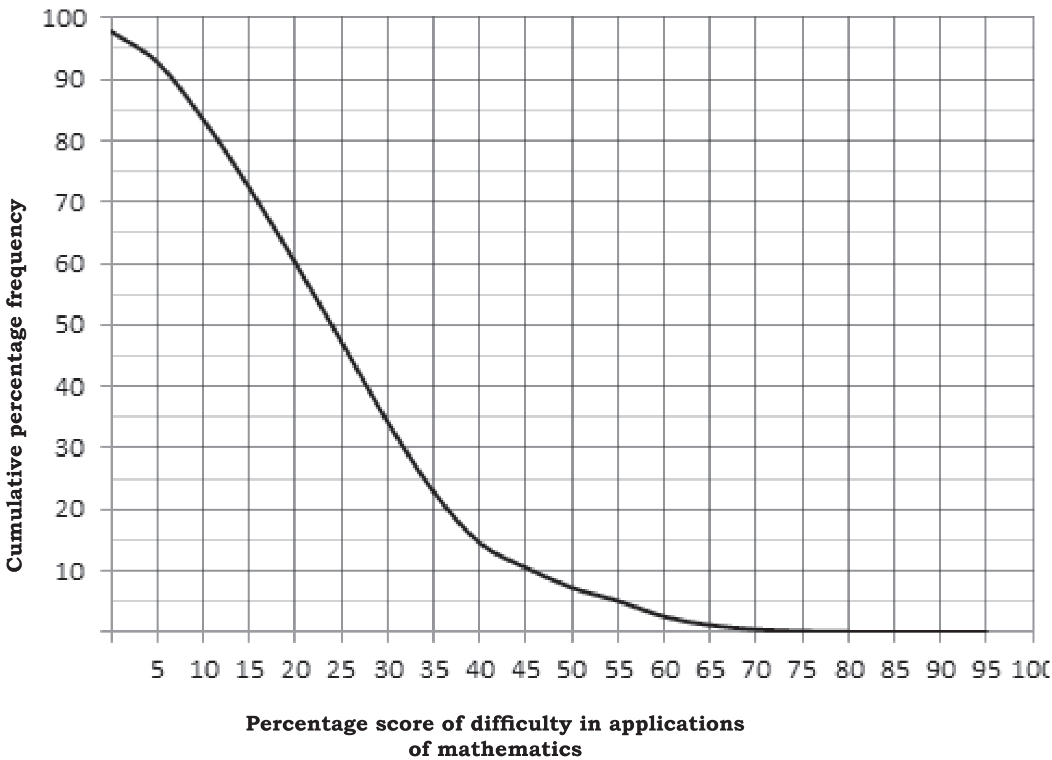


Fig. 6: Cumulative percentage frequency curve of difficulty in application of mathematics

Table 6: Mean Percentage Scores of Difficulty in Application of Mathematics in Each Concepts

S. No.	Physico-mathematical Concepts	Mean percentage score of difficulty in application of mathematics
1.	Acceleration	41.02
2.	Displacement	35.21
3.	II Equation of Motion	34.51
4.	Speed	29.90
5.	Law of Conservation of Momentum	25.60
6.	Distance	22.63
7.	Velocity	21.11
8.	Newton's Second Law of Motion	10.11
9.	III Equation of Motion	2.34

From Table 6, it is clear that the students possess high level of difficulty in application of mathematics in the topics 'Acceleration', 'Displacement', 'II Equation of Motion' and 'Speed'; moderate level of difficulty in application of mathematics in the topics 'Law of Conservation of Momentum', 'Distance', 'Velocity' and 'Newton's Second Law of Motion'; low level of difficulty in application of mathematics in the topic 'III Equation of Motion'.

Results

The above findings help us to understand that among the higher secondary science students, the extent of Conceptual Difficulty in 'Motion' is high whereas, the extent of Physico-mathematical Conceptual Difficulties in total and in the categories, viz., Creating or Identifying the Formula, Extracting Information from Diagrams or Graphs, Creating Schematic Diagrams or Graphs and Application of Mathematics, are moderate.

The difficulty faced by students for each concept differs from category to category. The ranking of concepts based on the extent of CD, PMCDs and its categories is given as Table 7.

Table 7 : Ranking of Concepts Based on the Extent of Difficulty in Each Category

Conceptual difficulty	Physico-mathematical conceptual difficulties	Difficulties in creating or identifying the formula	Difficulties in extracting information from diagrams or graphs	Difficulties in creating schematic diagrams or graphs	Difficulties in application of mathematics
III Equation of Motion	Displacement	Displacement	Acceleration	Velocity	Acceleration
II Equation of Motion	Acceleration	Newton's Second Law of Motion	Displacement	Displacement	Displacement
Law of Conservation of Momentum	II Equation of Motion	Velocity	Distance	Newton's Second Law of Motion	II Equation of Motion
Velocity	Velocity	Acceleration	Speed	Acceleration	Speed
Speed	Distance	Distance	Newton's Second Law of Motion	Distance	Law of Conservation of Momentum
Newton's Second Law of Motion	Newton's Second Law of Motion	II Equation of Motion	Velocity	Speed	Distance
Distance	Law of Conservation of Momentum	Law of Conservation of Momentum			Velocity
Acceleration	Speed	Speed			Newton's Second Law of Motion
Displacement	III Equation of Motion	III Equation of Motion			III Equation of Motion

Discussion

It is quite pathetic that the students who had encountered with the basic concepts of 'Motion' in their high school Physics, does not possess the basic level of conceptual understanding in those topics even in their higher secondary stage. Even if the higher secondary science stream students with conceptual understanding are taken into consideration, the Physico-mathematical conceptual understanding is only moderate. The extent of PMCD in four categories is different for different topics. However, the extent of PMCD in displacement is comparatively high in all the categories. These findings prove that it is the need of the hour to rethink upon the various aspects regarding the transaction of concepts from mechanics and problem solving involving mathematical operations in the secondary and higher secondary stages of schooling.

Conclusion and Implications

Conceptual understanding is based upon building of the higher order thinking capacity. If the student lacks proper conceptual understanding, he or she will never be able to apply it to solve problems based on the concept. So, building the schemas of the conceptual framework is very important to learn any of the subject areas. In physics, distance, displacement, speed, velocity, acceleration, equations of motion, Newton's laws of motion and Law of conservation of momentum are the grass root level concepts introduced in the Class VIII upon which the whole idea of mechanics is built up which they encounter in their higher classes.

Physics teachers are meant to identify the basic conceptual foundations in students regarding the topics taught in high school stage, before teaching the theories and principles in higher secondary physics. Otherwise, it will hinder the vertical transfer of knowledge among learners. If the teacher finds gaps in the basic conceptual understanding in 'Motion along a straight line' among students at higher secondary level, adequate remedial assistance must be provided to them, before introducing 'Motion in a three-dimensional space'.

Visualisation of concepts is very necessary especially in learning mechanics. The various representations used by teachers to transact the topics helps in deepening the understanding among students. The method of using representations is considered to be a powerful instructional strategy especially for physics teachers, who have to deal with the students with low level of conceptual understanding and mathematical problem-solving ability in mechanics. A single concept has to be presented to the learners via different modes or representations like, visual, verbal, graphical, mathematical, etc. The results of this study are strong evidence that the ability of students to translate or to switch from one type of representation to other, for example, from verbal representation to diagrams or graphs and vice-versa, is low.

At high school level, while introducing the basic concepts in motion, the teacher has to use representations including computer based ones such as animations and voice over simulations which discuss the instances from daily life, where each learner can personally connect and the applications of these concepts occur. For example, instead of teacher explanations and blackboard drawings, showing the video of the school and the various roads that connect home and school, the teacher can explain that the shortest path corresponds to displacement in a more effective manner. Then the students will be able to visualise the disparity between distance and displacement. This example can also be elaborated with the situations of students who actually take different roads to reach the school. The same is in the case of other basic concepts discussed in this paper.

To tackle the difficulties in dealing with mathematical problem solving in 'Motion', equal weightage and importance as that of concept or theory learning should be laid down in problem-based exercises in the classroom, from high school stage onwards so that students become more familiarised with application of mathematics in solving mechanics problems. Formative assessments must be strengthened by providing more problem-solving drills, exercises, home works and assignments that focuses mainly on practising the transformation from one representation to another.

To cater to the mathematical skill of the students, they are to be monitored strictly and systematically in each and every step in problem solving so as to derive at the precise solution to the mathematical physics problems. Sufficient attention should be directed to foster the ability of the student to plan for problem solving and to execute it efficiently. Specifically, while teaching problem solving in 'Motion', students should be taught first how to convert the questions to symbolic representations using alphabetical notations and diagrams, and only then to proceed with pure mathematical simplifications and calculations to reach the solution.

Physics teachers can plan the instructions regarding the conceptual areas in 'Motion', while the mathematics teachers can involve completely in the procedural skills such as reading and construction of graphs, mathematical figures and diagrams and simplification of equations. The weaknesses of the pupil in executing mathematical procedures like solving equations and geometrical problems have to be pinpointed in the remedial sessions in mathematics.

In Kerala, the situation is that the higher secondary school teachers are already burdened with the exhaustive subject area to be taught to a wide student group in a comparatively less span of time. In such circumstances, the distribution of simulations and interactive multimedia packages on the concepts in 'Motion' along with the mathematical practice exercises and worksheets on MOODLE platform will be a boon for all the students who find it difficult to learn solving of problems in a logically systematic way.

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