Mathematical Thinking and Teaching Practice of School Teachers at Upper Primary Level

Ashutosh Prabhakar* and Ittira Poovaiah Gowramma**

Abstract

One of the goals of teaching mathematics is to develop mathematical thinking among students and to be able to identify, where mathematics is applicable in the real world. To inculcate mathematical thinking among students, it is necessary to involve mathematical thinking activities as a prominent part of their education. And if teachers are to encourage mathematical thinking in students, they need to engage in mathematical thinking throughout the lesson themselves. The crucial and essential part of the current scenario is knowing about teachers' thinking towards their teaching practice. The effect of teachers' mentality on improving the teaching process seems to be neglected to some extent. Considering this, existing gap, this study aims to go through teachers' minds and find out how they think. A purposive sampling technique was used to select ten mathematics teachers teaching at upper primary level in five Kendriya Vidyalayas in Patna region. Open-ended questionnaires, rating scales and classroom observations were used to collect data, and analysed quantitatively and qualitatively to reveal the findings. Data formulation and thematic organisation were done through content analysis for further interpretation, which revealed that teachers with greater mathematical thinking skills employ more underlying mathematical processes and methods in their teaching such as analysing the subject matter, designing lessons for a specific goal, and understanding students' reactions and responses.

Keywords: *Mathematical Thinking, Teaching Practice, School Education and Upper Primary Teacher*

**Research Scholar*, Department of Teacher Education, Central University of South Bihar– 824236. (E-mail: ashuprabhakar.007@gmail.com)

***Professor*, Regional Institute of Education, Bhubaneswar–751022. (E-mail: gowriip@gmail.com)

Introduction

What is the primary purpose of mathematics education at the elementary and secondary levels? Simply said, the mathematisation of the child's mental processes. In the words of Wheeler (1982), "knowing how to mathematise is more beneficial than knowing a lot of mathematics." Polya (1969) said that the higher goal of math is to help a child develop inner resources, clear thinking and the ability to follow assumptions to logical conclusions, systematically solve math problems with the right attitude and deal with abstraction. Every student has the right to a high-quality mathematical education, as mathematics is essential. It is a practical, exciting, and creative study area that all students can appreciate and enjoy. It helps them develop their ability to solve problems and reason logically. It makes students curious and energetic to explore, and make sense of their world. Day-by-day, educators have grown in the importance of thinking in the educational disciplines. Today's students live in a time when thinking is more critical in mathematics than the skills, we ask them to practise repeatedly.

To develop the right attitude towards mathematics among students, teachers play a crucial role. Teachers want their students to have good ways of thinking that they stick to allowing them to think quickly and easily. Teachers must adapt to enhance students' learning and motivate them (Adam, 2004). So, research (Chapman, 2017) still needs to pay attention to how teachers think and how that relates to how well they teach. In the equation for education, how teachers think is a significant factor that affects how they teach and what they do in the classroom. It helps teachers adopt a technique of making sense of what, why and how students learn. Because of this, teaching math requires understanding what teachers think about the math they teach, how they teach it and how their students learn. This requires focusing not on what teachers should do but on what they are already doing and why. To meaningfully support teachers' knowledge also requires an understanding of their thinking. Most researchers have looked at how students' thinking styles affect their academic performance in the classroom (Zhang, 2002; Sternberg, 1997) but few have looked at teachers' thinking styles at the school level from the point-of-view of an administrator. Therefore, this research is dedicated to investigate teachers' thinking styles in mathematics teaching and learning to increase their knowledge in these areas of mathematics education.

Teachers' Thinking about Mathematics Teaching-Learning

Academic conversations often talk about how much teachers know about the subjects they teach but rarely talk about how they think (Sullivan, 2003). Goe et al. (2008) said that a teacher's idea of teaching and how they do it are affected by how they think, how much experience they have, how well they know the subject matter and how they teach. From all of them, thinking is a major one that impacts teachers' teaching practice, planning, decision-making and subsequent classroom behaviour. Similarly, Fullan (1991) pointed out, "Educational change depends on what teachers do and think". It is as simple and as complex as that. "Educational reform becomes complicated because teachers' work is greatly influenced by their thoughts" (Clark and Peterson, 1986). It is seen that "there is so much reform but so little change occurs" (Payne, 2008). Many educational reforms, policies and strategies fail because they do not delve deeply into teachers' minds (Fullan, 2009). So, educational reform is only possible when teachers' ways of thinking are to be considered.

In general, thinking is a person's prior knowledge and understanding, which involves the critical and creative aspects of the mind. In our daily life thinking processes, there is much reasoning going on in our minds when an operation is performed, problem-solving, reasoning, critical thinking, creative thinking, reflective thinking, and so on. At this stage, individuals must organise their thinking systems well to get meaningful results. (Fisher, 2005; Ersoy and Basar, 2012; Gunes, 2012).

However, Messick (1976) pointed out that 'thinking' is a term used in cognitive psychology to describe the consistent individual difference in the preferred way of organising and processing information and experience. In the same way, a person's thinking style is how they prefer to deal with information and tasks (Sternberg, 1997; Zhang and Sternberg, 2005; Hunt, 2008).

Mathematics as a discipline is one of the important branches of science that requires a particular way of thinking (Maddox, 2002). It helps us learn skills we can use in real life, like thinking, making connections between events, reasoning, calculating, doing math, and so on (Umay, 2003). These skills support each other to make things meaningful. A mathematical thinking style provides different ways for individuals to present, understand, and think through the facts and connections of mathematics (Ferri, 2006). Mathematical

thinking requires physical, mental, and emotional connections to understand ourselves and the world (Hudson et al., 2015). It is said that mathematical thinking is a kind of thinking that is realised not only in cases with numbers and abstract concepts of mathematics but also in our daily life (Yesildere and Turnuklu, 2007). Different frameworks have been proposed for studying mathematical thinking. Like, Torner and Pehkonen (1999) have proposed the following perspectives:

- **Traditional Perspectives:** Mathematics is a set of skills like calculations, rules, formulas and procedures.
- **Formalist Perspectives:** Mathematics is viewed as logic that has proofs, concepts and precise language.
- **Constructivist Perspectives:** Mathematics is seen as developing thought processes by actively engaging, and building rules, regulations and formulae from real experience.

Rationale of the Study

Research on the mathematics teacher and initiatives to transform mathematics education place significant emphasis on teacher cognition, and its relationship to effective teaching. The teacher's way of thinking is crucial to the educational equation. It has a significant impact on teachers' behaviour and students' classroom experiences. It represents a teacher's approach to making meaning of what they do, why they do it and how they do it. Therefore, to comprehend mathematics education from a classroom or practice-based perspective, it is necessary to understand mathematics teachers' perspectives on the mathematics they teach, their pedagogical approaches and their students' learning. This needs a focus on what teachers are actually doing and why, as opposed to what they should be doing. Understanding how to genuinely assist teachers' learning involves understanding and comprehending their thinking patterns (Chapman, 2017). Mathematical thinking is not just reasoning about the subject matter of mathematics but a style of thinking about particular operations, processes and dynamic ideas (Burton, 1984). Different aspects have emerged in mathematical thinking and teaching practice. Argyle (2012) emphasised that sense-making is the active process of mathematical thinking. It is a constant, cyclical way of thinking in which a person tries to make sense of a massive amount of sensory information. On the other hand, Mordan and Pourasadollah (2014) pointed out that teachers' teaching

methods and personal experiences are more important than what they have learned in school. National Curriculum Framework (NCF, 2005) emphasised that schools should focus on the constructivist approach and learner-centred pedagogy to improve mathematical thinking. However, Khalid (2006) emphasised that more effort is needed to train the teacher to be aware of the importance of mathematical thinking to make it a success and an available avenue to implement lesson study for teachers nationwide. On the other hand, most of the research has focused on teachers' abilities and knowledge (Buehl and Fives, 2009); the role of teachers in the classroom (Wan et al., 2011); the areas of evaluation (He et al., 2012); and the role of technology in the classroom (Hermans et al., 2008). However, teachers' thinking and teaching practices seem to be somewhat neglected. This gap in educational planning needs more research so that the results can be used to pave the way for more efficient decision-making and planning. The researcher tried to analyse teachers' thinking and teaching practices from different aspects, and the study's results paved the way for quality teaching practise.

Objectives of the Study

The study aimed to examine upper primary school mathematics teachers' mathematical thinking and teaching practices. Hence, the objectives of the study were:

1. To study mathematics teachers' mathematical thinking and teaching practices in the classroom context.
2. To study the mathematics teaching approaches used by teachers to elicit mathematical thinking among students in the classroom context.
3. To study the assessment and evaluation strategies used by teachers to evaluate the students' responses.

Research Questions

1. What is the mathematics teachers' thinking about the way mathematics should be taught?
2. How the teaching approaches used by teachers to teach mathematics in the classroom elicit mathematical thinking among students?
3. Which different evaluation strategies do teachers employ to evaluate students' responses?

Research Methodology

Method

The descriptive survey method was adapted to investigate the mathematical thinking of school teachers about the teaching-learning practise of mathematics. First, an open-ended questionnaire was administered on different mathematics dimensions to elicit teachers' mathematical thinking. A rating scale was used to get the participants' thoughts on teaching practice. It was based on different dimensions of mathematics. Furthermore, the classroom teaching processes were observed to reveal the teachers' mathematics teaching practices.

Population and Sample

The mathematics teachers of upper primary level teaching in Kendriya Vidyalaya in Patna Region, India, constituted the population of the study. Researchers used purposive sampling technique as the data had to be collected within a stipulated time period as well as to know certain in-depth characteristics from a particular subset of population of the study. Ten mathematics teachers teaching at the upper primary level were selected from five Kendriya Vidyalayas in Patna region for the study.

Instrument

A Mathematical Thinking Scale (MTS) for teachers was developed. The scale is divided into sections A, B and C. In section A, participants were asked for demographic information. In section B, open-ended questions and a questionnaire based on a four-point rating scale were developed on the dimensions of mathematics: the nature of mathematics, the teaching of mathematics, learning of mathematics, mathematics assessment and evaluation from participants. Finally, in section C, the Classroom Observation Schedule (COS) was used to observe the actual mathematics teaching-learning process. In the process of the tool construction, open-ended questions (8 out of 12) and (24 out of 35 items) for Likert's scale were finalised for try-out after getting feedback from the experts.

Reliability of the Tool

The reliability of the tool was calculated through Cronbach's alpha (Coefficient alpha) and along with their dimensions which are mentioned in Table 1.

Table 1 Cronbanch's Alpha for Dimensions of Mathematical Thinking Scale (MTS) and its Dimensions

Dimensions of Mathematical Thinking Scale (MTS)	Cronbach's Alpha Coefficient	Qualitative Descriptor
Nature of Mathematics	0.708	Acceptable
Teaching of Mathematics	0.790	Acceptable
Learning of mathematics	0.753	Acceptable
Mathematics Assessment and Evaluation	0.862	Acceptable
Mathematical Thinking Scale	0.912	Great

Reliability coefficient based on internal consistency of the entire scale was found 0.912, indicating the scale is highly reliable in nature.

Validity of the Tool

The opinions of experts determined the content validity of the scale. Four experts from the Regional Institute of Education (NCERT), Bhubaneswar: one each from the mathematics discipline, linguistic discipline, from psychology and education. Following expert comments and feedback, minor modifications were made to the scale. As a result, eight open-ended and 24 items for the rating scale were determined to be suitable for the final tool.

Data Collection Procedure

First, permission letter for the concerned authorities of the schools was taken from the Regional Institute of Education (NCERT), Bhubaneswar for the permission to collect data from the teachers and observe the classrooms. Then, permission from the concerned school authorities was taken to collect data from the participants. Participants were informed about the purpose of the study before administering the tool. After developing rapport with the participants, all essential instructions were stated clearly in the tool. After data collection, the scoring process was completed.

Findings of the Study

Response of the participants to the open-ended questions

Teachers responded to and interpreted the questions in several ways. Some teachers interpret the questions in terms of their

primary mathematics learning in the classroom, and some in the purpose of mathematics for them in everyday life 'here and now'. Some interpreted in terms of their long-term goals. The following themes emerged from the participants' responses to the open-ended questions which are mentioned in Table 2.

Table 2 Theme-wise description and examples from the open-ended responses of the participants

Themes	Description of themes	Examples
Everyday Life Here and Now	The usefulness of mathematics for everyday life is to work with daily life.	Figuring the total amount of money needed for work; accurately measuring lengths, widths and angles, and estimating project costs, etc.
Problem-solving	It helps to solve problems that arise in daily life, in mathematics and other contexts.	Gathering, organising, interpreting and communicating information to solve problems.
Thinking	Mathematics is all about thinking or using the brain.	When a mother prepares food, they never calculate the amount of sugar or salts but thinks of it.
Learning	Mathematics is a process of determining logical methods.	Once the children learn or observe the way of doing the activities, they follow the same procedure and repeat it regularly.
Process Skill	Mathematics helps in making better connections between mathematical problems and concepts.	In the mall, we frequently see flat 50% off, buy one get one free and so on. So we estimate the quality and quantity, weight and price per unit, discount calculations and finally, buy it.

Table 2 shows that teachers give priority to the utility of mathematics and consider mathematics as a tool to address societal issues. It works as a background in every activity and helps solve real-life problems.

Table 3 Frequency and Percentage of Mathematics Teachers' Thinking and Practices on the Nature of Mathematics.

Perspectives	Description	Strongly Agree	Agree	Disagree	Strongly Disagree
Traditional	Mathematics is mainly a fixed body of knowledge, facts and procedures.	3(30%)	5(50%)	2(20%)	0(0%)
	Mathematics help me to understand concepts, principles and strategies.	5(50%)	4(40%)	1(1%)	0(0%)

Constructivist	Mathematics is all about problem solving and logical thinking.	4(40%)	3(30%)	3(30%)	0(0%)
	Mathematics is used in the real world.	8(80%)	2(20%)	0(0%)	0(0%)
Mix of traditional and constructivist	Mathematics is related to the physical world and involves thinking.	5(50%)	3(30%)	2(20%)	0(0%)
	Mathematics is partially a fixed body of knowledge, rules and procedures.	6(60%)	3(30%)	1(10%)	0(0%)

Mathematics teachers' ideas regarding the nature of mathematics were taken through a Likert scale questionnaire that asked them to respond with 'strongly agree', 'agree', 'disagree' or 'strongly disagree'. Table 3 shows the responses of the participants. In view of the traditional way of teaching, the majority (80 per cent to 90 per cent) of the teachers think that mathematics is a discipline with fixed knowledge of facts and procedures, and that it helps us understand different concepts, principles and strategies, respectively. According to the constructivist way of teaching, 70 per cent to 90 per cent of teachers thought that math is used in everyday life to solve problems by using different ways of thinking logically. Similarly, in the context of both views of teaching, most teachers (80 per cent to 90 per cent) use both modes of teaching in the physical world with rules and regulations. As supported by Cross (2009) and Thompson (1984), most teachers do not have one fixed framework for teaching. Instead, they have a mixture of constructivist and traditional views (Askew et al., 1997).

Thinking about Teaching of Mathematics

In this subscale, different teaching strategies, types and ranges of teaching actions, classroom activities and resources used in their teaching practice were explored with a rating scale, which asked them to respond with 'strongly agree', 'agree', 'disagree' or 'strongly disagree' to the statements.

Table 4 summarises their responses and shows that 80 per cent to 90 per cent of the teachers agreed that mathematics textbook is necessary for teaching and that ideas need to be explained in-depth before students are given mathematics problems. Similarly, the majority of the teachers (70 per cent to 90 per cent) agreed that

a variety of materials to be used during the teaching-learning process to facilitate students' understanding so that they can link new knowledge with real-life experience by exploring mathematical ideas. This thinking is consistent with the higher aim of mathematics envisaged by the NCF-2005. However, most of the people who answered (90 per cent) agreed that an activity-based learning environment should be set up to get students to look into problems and find solutions on their own.

Table 4 Frequency and Percentage of Mathematics Teachers' Thinking and Practices on the Teaching of Mathematics

Perspectives	Description	Strongly Agree	Agree	Disagree	Strongly Disagree
Traditional	A teacher should explain the mathematical rules and procedures before giving them mathematical problems.	6(60%)	3(30%)	1(10%)	0(0%)
	A teacher should follow students' textbooks in giving instructions and make sure students receive this knowledge.	5(50%)	3(30%)	2(20%)	0(0%)
Constructivist	A teacher should help students link their new knowledge to their real-life experiences.	5(50%)	2(20%)	3(30%)	0(0%)
	A teacher should use a variety of materials to help students understand and give them things they can play with to learn more about math concepts.	6(60%)	3(30%)	1(10%)	0(0%)
Mix of traditional and constructivist	A teacher should provide activities that encourage students to engage actively.	7(70%)	2(20%)	1(10%)	0(0%)
	A teacher should provide students with problematic situations to investigate and find solutions.	6(60%)	3(30%)	1(10%)	0(0%)

Mathematics teachers' views about mathematics learning, which include behaviour, learning activities as well as mental activities, were explored using a rating scale which asked them to respond with 'strongly agree', 'agree', 'disagree' or 'strongly disagree' with the statements are depicted in Table 5.

Table 5 Frequency and Percentage of Mathematics Teachers' Thinking and Practices on the Learning of Mathematics

Perspectives	Description	Strongly Agree	Agree	Disagree	Strongly Disagree
Traditional	Students learn from teachers' explanation with demonstration.	5(50%)	2(20%)	3(40%)	0(0%)
	Learning is enhanced when students explain and demonstrate their solutions to others.	7(70%)	3(30%)	0 (0%)	0(0%)
Constructivist	Students' learn by actively engaging in learning activities and doing real-life related activities.	6(60%)	2(20%)	2(20%)	0(0%)
	Doing repetitive practice for mastery of skills is important.	3(30%)	2(20%)	5(50%)	0(0%)
Mix of traditional and constructivist	Listening to teachers' explanations and repeating practice is essential in learning mathematics.	7(70%)	2(20%)	1(10%)	0(0%)
	Students' prior experience plays an essential role in learning.	6(60%)	3(30%)	1(10%)	0(0%)

From one point-of-view, students learn mathematics best when their teachers show them how to do it (70 per cent) and when they can show others how to do it (100 per cent). Furthermore, active participation in various activities accelerates the learning process (80 per cent) and repetitive practices aid in skill mastery.

However, most teachers (90 per cent) believed that students' prior experiences play an essential role in understanding the teacher's explanation and mastery of skills.

Mathematics Assessment and Evaluation

In this subscale, different assessment and evaluation strategies were explored with the help of a classroom observation schedule and rating scale, which asked them to respond with 'strongly agree', 'agree', 'disagree' or 'strongly disagree' with the statements.

Table 6 Frequency and Percentage of Mathematics Teachers' Thinking and Practices on Mathematics Assessment and Evaluation

Perspectives	Description	Strongly Agree	Agree	Disagree	Strongly Disagree
Traditional	Written examinations	8(80%)	1(10%)	1(10%)	0(0%)
	Verbal questions	1(10%)	1(10%)	8(80%)	0(0%)
Constructivist	Assignment	5(50%)	3(30%)	2(20%)	0(0%)
	Problem-solving session	1(10%)	2(20%)	7(70%)	0(0%)
Mix of traditional and constructivist	When students make errors, the best remedy is to make them repeatedly practice these types of problems.	6(60%)	2(20%)	2(20%)	0(0%)
	Mathematics assessments make the goals for learning objectives for students.	7(70%)	2(20%)	1(10%)	0(0%)

Table 6 shows an analysis of teachers' answers about how they test and grade math students with different themes to a different degree. However, respondents give more priority (90 per cent) to written examinations than verbal questioning to assess students' understanding levels. Similarly, assignments are given more credit (80 per cent) than coordinating problem-solving sessions (30 per cent). In another context, most teachers (80 per cent) agreed that repeating similar types of problems is the best way to eliminate errors.

Theme-wise description of teachers' responses during the classroom observation

Based on the classroom observation certain useful techniques used by teachers are identified. They are briefly described under broad themes as given below.

Buying Time

When something remains unclear to students during explaining, teachers pause, smile, repeat the statement and ask the students, 'What does anyone else think about that?' Teachers use this technique to get additional information about students' thinking and responses to help them move the class forward.

Understand Their Process

During the class, the teacher sometimes asks the students to come to the blackboard to solve problems. When students solve the problem step-by-step, they understand how to get the correct answer and why they perform specific steps to get the answer. The students understand both how to solve a problem and why they perform specific steps in the process of solving a problem.

Catch Their Mistakes

Teachers formulate formative questions and ask the students to explain them. While explaining the answer, students catch the mistakes they made by judging the teacher's facial expression and response. This allows students to explain the mistake and why it was wrong. Students learn the most when they catch their mistakes and avoid repeating them in future problems.

Generalise the Steps for the Next Problem

A significant component of math is recognising patterns. The teacher left out three to four steps, while explaining the problems on the board and asked the students to think. Students try to find patterns and use them to solve problems that are similar but more complicated.

Engage Every Student

Teachers give problems on the board and ask them to solve them in their notebooks. They then move bench-by-bench to see the students' responses. By doing this, they try to engage all the students in the pace of learning.

It was also noticed that teachers try to figure out how much the students already know. They try to set learning goals that can be reached, choose teaching methods based on students' cognitive levels, plan and organise teaching activities from students' points-of-view, and change their teaching based on how well the students are learning. Hence, all of these activities should work complementarily to promote thoughtful and intentional planning for subsequent lessons.

Analysis and Discussion

Mathematics teachers' thinking about the way mathematics is to be taught

- Most mathematics teachers think that students should be encouraged to explore, investigate and create their knowledge through various activities.
- During classroom activities, teachers try to get information about students, like how they learn and interact, what they appear to know and what interests them.
- Teachers think that students should be given equal opportunities to think and work quietly by themselves.
- They think students should be given full opportunities to work independently and collaboratively to make sense of ideas.

Classroom practices in teaching and learning mathematics

- Teachers think students should be given opportunities to struggle with ideas to develop and use mathematical processes for example, justification, abstraction and generalisation.
- The teaching strategies included verbal explanation interspersed with question-answer sessions in the classroom. Occasionally, teachers explain, demonstrate the activities and organise students in small groups to solve the problems.
- They think students need to connect with their existing mathematical knowledge and understanding to make sense of a new concept or skill. Students grasp the connectivity of diverse mathematical ideas, and the linkages between mathematics and the real world by completing tasks. When students have the opportunity to use mathematics in real-life situations, they learn about its importance and contribution to other fields of knowledge.

- Some teachers' views are authoritative as they think mathematics classrooms should be initiated as per the teachers' instructions. However, most of them contradict and give more importance to students' freedom in school.

Mathematics teachers' knowledge to facilitate students' learning in the classroom

- They think interacting with students, encouraging discussion and cooperative learning can help in improving students' creativity.
- Most of the teachers' reflected that paying attention to individual students and understanding prior knowledge or misconceptions help to maximise students' learning potential.
- Most mathematics teachers think mathematics is best learned through exploring and investigating mathematical ideas.

Teachers Questioning

We paid close attention to the teachers' follow-up questions meant to expand on the students' initial responses. Frequently, these questions were designed to get students to explain things more clearly to figure out why they made mistakes, to get more information on how to solve problems or to draw attention to key mathematical ideas. Four types of teacher questioning techniques were identified and examined as a result of our iterative research process; general questions, particular questions, probing sequences of specific questions and leading questions. There was no connection between general enquiries and anything a student explicitly mentioned. Specific enquiries focused on a particular aspect of a student's explanation. Probing sequences of specific questions contained numerous instructor enquiries and multiple students' replies. They were composed of more than two related enquiries regarding a particular statement made by a student. The teacher allowed students to react, while directing them toward specific explanations or replies through leading questions. Other approaches that teachers employed to make student thinking apparent included:

- Re-voicing or repeating student answers or explanations.
- Outlining strategies they felt students used to solve specific problems.
- Emphasising mathematical principles in students' explanations.

Conclusion

The present study emphasised teachers' mathematical thinking and teaching practices in the classroom context. How teachers set up classroom instruction depends on their knowledge, thinking skills, experience, and understanding of mathematics teaching and learning. They need thinking skills that help them to recognise and then act upon the teaching opportunities that come up without warning. Even though teachers make decisions based on what students think in class, Clarkson and Presmeg (2008) say that teachers should also learn how students think outside of class. Teachers with better mathematical thinking skills analyse the subject matter, plan lessons with a specific goal and figure out what the students are trying to say by their answers. This view is also supported by Stacey (2006), who concluded that mathematical thinking is essential in every step in organising effective lesson plans for a better teaching-learning process. Mathematical concepts are seen as static, fixed, either discovered or waiting to be discovered and socially constructed phenomena (Ernest, 1989; Amirali and Halai, 2010).

Learning mathematics is a solitary process that involves goals, plans, ideas and mental pictures of different ways to teach (Ernest, 1989). In planning classroom experiences, a mathematics teacher must balance these ideas as mathematics is seen both as a social phenomenon and a solitary process. Therefore, from the beginning of teacher training, teacher education programmes must provide opportunities for aspiring teachers to reflect on their mathematical thinking and teaching practices. This may be the solution to reducing the discrepancy between thinking and practices, and enhancing teaching practice.

Educational Implications

The study has significant educational implications for school education. It will raise the educational standard of mathematics teaching by enlightening the teaching process and making an effort to improve teaching practice. The result can pave the way for better education planning, course materials, curriculum changes and teaching practices.

Limitations

The research study had some methodological limitations. First, it is difficult to generalise the study results due to the small sample size. Second, the study includes qualitative and quantitative responses, which the researcher could not link due to an anonymous response.

References

Adam, S. 2004. Ethno-Mathematical Ideas in the Curriculum. *Mathematics Education Research Journal.* 16(2). 49–68.

Amirali. M. and A. Halai. 2010. Teachers' Knowledge about the Nature of Mathematics: A Survey of Secondary School Teachers in Karachi, Pakistan. *Bulletin of Education and Research.* 32(2). 45–61.

Argyle, S. F. 2012. Mathematical Thinking: from Cacophony to Consensus. Kent State University College and Graduate School of Education, United States of America.

Askew, M., M. Brown, V. Rhodes, D. Wiliam and D. Johnson. 1997. Effective Teachers of Numeracy in Primary Schools: Teachers' Beliefs, Practices and Pupils' Learning. Paper Presented at the British Educational Research Association Annual Conference, University of York.

Buehl, M. M. and H. Fives. 2009. Exploring Teachers' Beliefs about Teaching Knowledge: Where Does it Comes from? Does it Changes? *Journal of Experimental Education.* 77(4). 367–407.

Burton, L. 1984. Mathematical Thinking: The Struggle for Meaning. *Journal for Research in Mathematics Education.* 15(1). 35–49.

Chapman, O. 2017. Attending to Mathematics Teacher Thinking. *Journal of Mathematics Teacher Education.* 20(1). 1–4.

Clarkson, P. and N. C. Presmeg. 2008. Critical Issues in Mathematics Education: Major Contributions of Alan Bishop. Springer. Dordrecht, Netherlands.

Clark, C. M. and P. L. Peterson. 1986. Teachers' Thought Processes. *Handbook of Research on Teaching.* 3. 255–296. Macmillan, New York.

Cross, D. I. 2009. Alignment, Cohesion, and Change: Examining Mathematics Teachers' Beliefs Structures and their Influence on Instructional Practice. *Journal of Mathematics Teacher Education.* 12. 325–346.

Ernest, P. 1989. The Knowledge, Beliefs and Attitudes of the Mathematics Teacher: A Model. *Journal of Education for Teaching.* 15. 13–33.

Ersoy, E. and N. Baser. 2012. The Development of Mathematical Thinking Scale. *Kastamonu Education Journal.* 21(4). 1471–1486.

Ferri, R. B. 2006. Mathematical Thinking Styles—An Empirical Study. *European Research in Mathematics Education.* 3. 1–9.

Fisher, R. 2005. Teaching Children to Think. Nelson Thornes. Cheltenham, United Kingdom.

Fullan, M. 1991. The New Meaning of Educational Change. Teachers College Press, New York.

———. 2009. The Challenge of Change: Start School Improvement Now. Corwin, California.

Goe, L., C Bell and O. Little. 2008. Approaches to Evaluating Teacher Effectiveness: A Research Synthesis. National Comprehensive Centre for Teacher Quality. Washington DC.

Gunes, F. 2012. Developing Students' Thinking Skills. *Turkish Science Studies*. 32. 127–146.

Hermans, R., J. Tondeur, J. Van Braak and M. Valcke. 2008. The Impact of Primary School Teachers' Educational Beliefs on the Classroom Use of Computers. *Computers and Education*. 51. 1499–1509. doi: 10.1016/j.compedu.2008.02.001

He, Q., M. Valcke and A. Aelterman. 2012. A Qualitative Study of In-service Teacher Evaluation Beliefs. *Journal of Educational Sciences and Psychology*. 2(2). 1–14.

Hudson, B., S. Henderson and A. Hudson. 2015. Developing Mathematical Thinking in the Primary Classroom: Liberating Students and Teachers as Learners of Mathematics. *Journal of Curriculum Studies*. 47(3). 374–398.

Hunt, E. 2008. Applying the Theory of Successful Intelligence to Education—the Good, the Bad, and the Ogre; Commentary on Sternberg et al. (2008). *Perspectives on Psychological Science*. 3(6). 509–515.

Khalid, M. 2006. Mathematical Thinking in Brunie Curriculum: Implementation, Issues and Challenges. University of Tsukuba, Japan.

Maddox, R. B. 2002. Mathematical Thinking and Writing: A Transition to Higher Mathematics. Academic Press. San Diego, California.

Messick, S. 1976. Personality Consistencies in Cognition and Creativity. In S. Messick (Ed.). *Individuality in Learning*. 4–23. Jossey-Bass, San Francisco.

Moradan, A. and R. Pourasadollah. 2014. *T*eachers' Thinking about their Teaching: A Critical Study on Iranian TEFL Teachers. *Procedia—Social and Behavioral Sciences. 98*. 1194–1203.

National Council of Educational Research and Training. 2005. National Curriculum Framework: National Focus Group on Teaching of Mathematics. NCERT, New Delhi.

Payne, C. M. 2008. So Much Reform, So Little Change: The Persistence of Failure in Urban Schools. Harvard Education Press. Cambridge, Massachusetts.

Polya, G. 1969. The Goals of Mathematical Education in Communicator. The Magazine of the California Mathematics Council.

Stacey, K. 2006. What is Mathematical Thinking and Why is it Important? University of Melbourne, Australia.

Sternberg, R. J. 1997. *Thinking Styles*. Cambridge University Press. New York, NY.

Sullivan, P. 2003. Editorial: Incorporating Knowledge of and Beliefs about Mathematics into Teacher Education. *Journal of Mathematics Teacher Education*. 6. 293–296.

Thompson, A. G. 1984. The Relationship of Teachers' Conceptions of Mathematics and Mathematics Teaching to Instructional Practice. *Education Studies in Mathematics*. 15(2). 105–127.

Torner, G. and E. Pehkonen. 1999. Teachers' Beliefs on Mathematics Teaching—Comparing Different Self-estimation Methods: A Case

Study. https://duepublico2.unidue.de/servlets/MCRFileNodeServlet/duepublico_derivate_00005246/mathe91999.pdf

Umay, A. 2003. Mathematical Reasoning Ability. *Journal of Teacher Education*. Hacettepe University. 24. 234–243.

Wan, W., G. D. Low and M. Li. 2011. From Students' and Teachers' Perspectives: Metaphor Analysis of Beliefs about EFL Teachers' Roles. *System*. 39(3). 403–415. https://doi.org/10.1016/j.system.2011.07.012

Wheeler, D. 1982. Mathematisation Matters. *For the Learning of Mathematics*. 3(1). 45–47.

Yesildere, S. and E. B. Turnuklu. 2007. Examination of Students' Mathematical Thinking and Reasoning Processes. *Journal of Faculty of Educational Sciences*. 40(1). 181–213.

Zhang, L. F. 2002. Thinking Styles and Cognitive Development. *Journal of Genetic Psychology*. 163(2). 179–195.

Zhang, L. F. and R. J. Sternberg. 2005. A Threefold Model of Intellectual Styles. *British Journal of Educational Psychology*. 17(1). 1–53.