FOUR PROBLEMS ON RATIO AND PROPORTION WITH THEIR SOLUTIONS

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Three jars respectively contain three different liquids whose weights are in some ratio. Three different powders each are mixed respectively with the liquids. Knowing the total weight of the liquids, that of the powders and that of the blended liquids, weights of the individual liquids and powders are determined. Another problem of ratio and proportion related to velocities of a train is set forth and solved. Finally two more problems of ratio and proportion pertaining to weights and price rates of three commodities are incorporated with their solutions.

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Introduction

Many textbooks of mathematics each, so to say, arithmetic or algebra from ninth to tenth standard contain a chapter on 'Ratio and proportion'. Also search on Google and YouTube to discuss this topic.

Problem 1

Three kinds of liquid designated by liquid 1, liquid 2, liquid 3 are in weight ratio $a_1{:}a_2{:}a_3$ Three kinds of powder designated by powder 1, powder 2, powder 3 are mixed in liquid 1, liquid 2, liquid 3, respectively. Then weights of the blended liquids are found to be in ratio $b_1{:}b_2{:}b_3$. The task is to find the weights of the individual liquids and powders while the total weight of the three liquids and that of the powders are A and B, respectively

Solution to the Problem 1

If x, y, z be the weights of the three liquids respectively and a, b, c be the weights of the three powders, respectively, then

$$\frac{x}{a_1}=\frac{y}{a_2}=\frac{z}{a_3}=\frac{x+y+z}{a_1+a_2+a_3}=\frac{A}{a_1+a_2+a_3} \qquad (1)$$

$$\frac{x+a}{b_1}=\frac{y+b}{b_2}=\frac{z+c}{b_3}$$

$$=\frac{x+y+z+a+b+c}{b_1+b_2+b_3}=\frac{A+B}{b_1+b_2+b_3} \qquad (2)$$

Hence, $x=\dfrac{Aa_1}{a_1+a_2+a_3}$, $y=\dfrac{Aa_2}{a_1+a_2+a_3}$,

$$z=\frac{Aa_3}{a_1+a_2+a_3} \qquad (3)$$

Using (2) and (3), we get

$$a=\frac{(A+B)b_1}{b_1+b_2+b_3}-\frac{Aa_1}{a_1+a_2+a_3}$$

$$b=\frac{(A+B)b_2}{b_1+b_2+b_3}-\frac{Aa_2}{a_1+a_2+a_3}$$

$$c=\frac{(A+B)b_3}{b_1+b_2+b_3}-\frac{Aa_3}{a_1+a_2+a_3} \qquad (4)$$

Problem 2

A train travels three different distances respectively with three different velocities in ratio a_1: a_2: a_3 and the average velocity of the train is found to be V_A. The corresponding times of travel are in ratio b_1: b_2: b_3.

Find the corresponding velocities of the train.

Solution to Problem 2

Let t_1, t_2, t_3 be the times during which the train travels with velocities v_1, v_2, v_3 respectively. Then

$v_1{:}v_2{:}v_3{:}\,{:}a_1{:}a_2{:}a_3$ (5)

$t_1{:}t_2{:}t_3{:}\,{:}b_1{:}b_2{:}\,b_3$ (6)

which can be rewritten as

$$\frac{v_1}{a_1}=\frac{v_2}{a_2}=\frac{v_3}{v_3}=\frac{v_1+v_2+v_3}{a_1+a_2+a_3} \quad (7)$$

$$\frac{t_1}{b_1}=\frac{t_2}{b_2}=\frac{t_3}{b_3}=\frac{t_1+t_2+t_3}{b_1+b_2+b_3}=\frac{T}{b_1+b_2+b_3} \quad (8)$$

where T is the total time of travel of the train. Multiplying (7) and (8), one gets

$$\frac{v_1t_1}{a_1b_1}=\frac{v_2t_2}{a_2b_2}=\frac{v_3t_3}{a_3b_3}=\frac{v_1t_1+v_2t_2+v_3t_3}{a_1b_1+a_2b_2+a_3b_3}$$
$$=\frac{V_AT}{a_1b_1+a_2b_2+a_3b_3} \quad (9)$$

where S = $v_1\,t_1+v_2\,t_2+v_3\,t_3$ is the total distance described by the train.

$$\mathrm{v}_1=\frac{V_ATa_1\,b_1}{t_1\left(a_1b_1+a_2b_2+a_3b_3\right)} \quad (10)$$

Eliminating t_1 and T between (8) and (10) is obtained

$$\mathrm{v}_1=\frac{V_Aa_1\left(b_1+b_2+b_3\right)}{\left(a_1b_1+a_2b_2+a_3b_3\right)} \quad (11)$$

Similarly,

$$v_2=\frac{V_Aa_2\left(b_1+b_2+b_3\right)}{\left(a_1b_1+a_2b_2+a_3b_3\right)}$$
$$v_3=\frac{V_Aa_3\left(b_1+b_2+b_3\right)}{\left(a_1b_1+a_2b_2+a_3b_3\right)} \quad (12)$$

which are found to be independent of the total time T of travel. The result can be verified by using (12) in (7).

Problem 3

Weights w_1, w_2, w_3, of three commodities are in ratio $w_1 : w_2 : w_3 :\,: a_1 : a_2 : a_3$ (13)

Their rates i.e., price per unit weight are respectively in ratio $R_1 : R_2 : R_3 :\,: b_1 : b_2 :$ b_3 (14)

The total cost and weight of the commodities are respectively C and W. Determine the price rates of the commodities.

Solution to Problem 3

$$\frac{w_1}{a_1}=\frac{w_2}{a_2}=\frac{w_3}{a_3}=\frac{w_1+w_2+w_3}{a_1+a_2+a_3}=\frac{W}{a_1+a_2+a_3} \quad (15)$$

$$\frac{R_1}{b_1}=\frac{R_2}{b_2}=\frac{R_3}{b_3} \quad (16)$$

Multiplying (15) and (16),

$$\frac{w_1R_1}{a_1b_1}=\frac{w_2R_2}{a_2b_2}=\frac{w_3R_3}{a_3b_3}$$
$$=\frac{w_1R_1+w_2R_2+w_3R_3}{a_1b_1+a_2b_2+a_3b_3}=\frac{C}{a_1b_1+a_2b_2+a_3b_3} \quad (17)$$

which give

$$R_1 = \frac{C\,a_1\,b_1}{(a_1\,b_1 + a_2\,b_2 + a_3 b_3)w_1} \quad (18)$$

$$R_2 = \frac{C\,a_2\,b_2}{(a_1\,b_1 + a_2\,b_2 + a_3 b_3)w_2} \quad (19)$$

$$R_3 = \frac{C\,a_3\,b_3}{(a_1\,b_1 + a_2\,b_2 + a_3 b_3)w_3} \quad (20)$$

Substituting the values of w_1, w_2, w_3 from (15) in (18), (19), (20) respectively one gets

$$R_1 = \frac{C\,b_1(a_1 + a_2 + a_3)}{(a_1\,b_1 + a_2\,b_2 + a_3\,b_3)W} \quad (21)$$

$$R_2 = \frac{C\,b_2(a_1 + a_2 + a_3)}{(a_1\,b_1 + a_2\,b_2 + a_3\,b_3)W} \quad (22)$$

$$R_3 = \frac{C\,b_3(a_1 + a_2 + a_3)}{(a_1\,b_1 + a_2\,b_2 + a_3\,b_3)W} \quad (23)$$

Problem 4 as an Extension of Problem 3

For selling purpose with profit, the price rates of the commodities are raised in ratio $c_1 : c_2 : c_3$ resulting in an increase in total cost of the three commodities by c (small). Find the enhanced price rates of the commodities and cost of the commodities after the increase in the rates.

Solution to the problem

If r_1, r_2, r_3, be the increases in price rates, we have

$$\frac{r_1}{c_1} = \frac{r_2}{c_2} = \frac{r_3}{c_3} = k \text{ (say)} \quad (24)$$

If R_1', R_2', R_3' be the enhanced price rates of the three commodities respectively, then

$$R_1' = R_1 + kc_1, \quad R_2' = R_2 + kc_1, \quad R_3' = R_3 + kc_1 \quad (25)$$

Using (25), enhanced cost of the commodities is given by

$$C' = w_1(R_1 + kc_1) + w_2(R_2 + kc_2) + w_3(R_3 + kc_3) \quad (26)$$

and

$$c = w_1 kc_1 + w_2 kc_2 + w_3 kc_3 \quad (27)$$

Or, $k = \dfrac{c}{w_1 c_1 + w_2 c_2 + w_3 c_3}$

Substituting for k from (27) and for R_1, R_2, R_3 from (21), (22) and (23) in (24), one gets

$$C' = w_1(R_1 + kc_1) + w_2(R_2 + kc_2) + w_3(R_3 + kc_3)$$

$$C' = w_1\left(\frac{cb_1(a_1 + a_2 + a_3)}{(a_1\,b_1 + a_2\,b_2 + a_3\,b_3)W} + \frac{c}{w_1 c_1 + w_2 c_2 + w_3 c_3} c_1\right) + w_2\left(\frac{cb_2(a_1 + a_2 + a_3)}{(a_1\,b_1 + a_2\,b_2 + a_3\,b_3)W} + \frac{c}{w_1 c_1 + w_2 c_2 + w_3 c_3} c_2\right) + w_1\left(\frac{cb_3(a_1 + a_2 + a_3)}{(a_1\,b_1 + a_2\,b_2 + a_3\,b_3)W} + \frac{c}{w_1 c_1 + w_2 c_2 + w_3 c_3} c_3\right) \quad (28)$$

References

Ratio and Proportion, Google Search.

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