PROBLEM-BASED LEARNING IN BASIC PHYSICS – VI

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In this article, sixth in the series, we present problems for a problem-based learning course from the area of quantum physics and nuclear physics. We present the learning objectives in this area of basic physics and what each problem tries to achieve with its solution.

Introduction

In this article, sixth in the series of 'Problem-based Learning in Basic Physics', we present problems on Quantum Physics and Nuclear Physics. Methodology and philosophy of selecting these problems are already discussed (Pradhan and Mody, 2009; Mody and Pradhan, 2011).

To review methodology in brief, we note here that this PBL (Problem-based Learning) starts after students have been introduced to formal structure of Physics. Ideally, students would attempt only the main problem. If they find it difficult, then depending upon their area of difficulty, right auxiliary problems have to be introduced by the teacher who is expected to be a constructivist facilitator. The teacher may choose as per her/his requirement or may construct questions on the spot to guide students to the right idea and method.

Problems on Quantum Physics

Learning Objectives

1. Exposure to quantum idea.
2. Planck and Einstein equation.
3. Bohr's theory of hydrogen atom and emission of spectral lines.
4. Calculation to get an idea of magnitude of quantities involved.

Problems

1. A beam of light has three wavelengths 4144 A°, 4972 A° and 6216 A° with a total intensity of 3.6×10^{-3} W/m^2 equally distributed amongst the three wavelengths. The beam falls normally on an area of 10 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in two seconds.
   - This problem needs to use photoelectric equation to find which of the wavelengths can induce photoelectric emission. It requires to calculate flux of photon (number of photons incident) of each wavelength to find the number of electrons emitted, by using Planck's quantum hypothesis.

   Tasks involved in solving this problem are:

   a. To find which of the wavelength is above threshold to be rejected.

b. To calculate number of photons incident corresponding to each wavelength using given information and Planck's quantum hypothesis.
c. To find number of electrons emitted as per Einstein's photoelectric equation.

2. Consider a potassium surface that is 75 cm away from a 100 W bulb. Suppose that the energy radiated by the bulb is 5% of the input energy. Treating each potassium atom as a circular disc of diameter 1 Å, determine the time required for each atom to absorb an amount of energy equal to its work function of 2.0 eV, according to the wave interpretation of light.
   - This problem involves estimation of time required for photoelectric emission on the basis of classical theory.
   - Since calculation of this problem contradicts observation, it illustrates the need for a new (quantum) theory.

   Tasks involved in solving this problem are:
   a. To calculate the amount of energy received by an atom per second from geometrical consideration from given data.
   b. To calculate the time using given data of energy required for emission of electron.

3. A hydrogen atom is excited from a state with n=1 to a state with n=3. Calculate (a) the energy absorbed by the atom, and (b) the wavelengths of the spectral lines emitted when the atom returns from n=3 to n=1 state. Display the lines emitted on an energy level diagram.
   - This problem involves simple calculation based on Bohr's theory and understanding of possible paths for de-excitation of electron.

   Tasks involved in this problem are:
   a. To calculate energy needed to excite H-atom from ground state to its second excited state.
   b. To recognise all the possible paths available to return to the ground state.
   c. To calculate wavelengths emitted in the process.
   d. To draw and show transitions on energy level diagram.

Problems on Nuclear Physics

Learning Objectives

1. To become familiar with the law of radioactive decay and nuclear energy.
2. Application of radioactivity.
3. Nuclear energy calculation. How important is it?

Problems

1. Assume when the planet earth was formed, U^{238} and U^{235} were present in equal abundance. At present, the ratio of their abundance is 140:1. If their half-lives are 4.5×10^9 years and 7.13×10^8 years respectively, estimate the age of the earth.

   Tasks involved in solving this problem are:
   a. To write equation for radioactive decay for each species of uranium.
   b. To estimate time lapsed assuming both the species were equally abundant initially using their present proportion.

2. A small quantity of solution containing Na^{24} radio nuclide (half-life 15 hours) of activity 1.0 μ curie is injected into blood of a person. A sample of the blood of volume

1cc taken after 5 hours shows an activity of 296 disintegrations per minute. Determine the total volume of blood in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person. (1 curie = 3.7×10^{10}dps) [JEE[1], 1993]

- This problem also involves simple exponential decay but this time for medical application.

Tasks involved in solving this problem are:

a. To understand here that the activity will vary not only with time but also with the sample of blood.

b. To estimate total volume of blood from the sample using exponential decay of activity and volume dependency.

3. In the interior of the Sun, a continuous process of 4 protons fusing into helium nucleus and a pair of positrons is going on. Calculate:

   i. The release of energy per process.

   ii. If the Sun radiates an energy of about 4×10^{26} J/s, how much mass gets converted into helium every second?

   iii. If mass of the Sun is 2×10^{30} kg, estimate the time it will take to convert all the mass into helium.

   iv. The Sun has been in its present stable state for 5×10^{9} yrs, how many more years will it continue to shine in this stable state? Neglect the energy carried away by neutrinos.

   Given: m $({}_1H^1)$ = 1.007825 *amu*,
   m $({}_2He^4)$ = 4.002603 *amu*,
   $m_e = m_\beta +$ = 5.5×10^{-4}*amu*;
   1 *amu* = 931.5 MeV

- This problem involves simple energy calculation in nuclear processes and estimation (order of magnitude) of life of the Sun.

Tasks involved in solving this problem are:

a. To calculate energy released per process.

b. To calculate mass of hydrogen consumed from solar luminosity.

c. To calculate the time required for Sun to burn all its hydrogen and hence estimate its life.

4. It is proposed to use the nuclear fusion reaction ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4$ in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with 25% efficiency in the reactor, how many grams of deuterium fuel will be needed per day [The masses of ${}_1H^2$ and ${}_2He^4$ are 2.0141 *amu* and 4.0026 *amu* respectively].

- This problem is similar to problem 3. Its importance is from the point of view of nuclear power generation.

Tasks involved in solving this problem are similar to those mentioned in problem 3.

Solutions

Quantum Physics

1. Photoelectric Effect

I = 3.6×10^{-3} W/m^2 distributed equally amongst three wavelengths

$\therefore$ Each will be 1.2×10^{-3} W/m^2 and W_0 = 2.3 eV

λ_1 = 4144 A°; corresponding energy $E_1 = hc/\lambda_1$ = 2.999 eV

[1] JEE — Joint Entrance Examination for admission to IIT.

λ_2 = 4972 A°; corresponding energy
$E_2 = hc/\lambda_2$ = 2.500 eV

λ_1 = 6216 A°; corresponding energy
$E_3 = hc/\lambda_3$ = 1.990 eV

Thus, only the first will be able to knock electrons off.

Thus, in 1 sec (from 10 cm²) λ_1 will be able to eject

$$N_1 = \frac{1/3}{hc/l_1} A = 2.5 \times 10^{12} \text{ electrons}$$

and λ_2 will be able to eject

$$N_2 = \frac{1/3}{hc/l_2} A = 6.0 \times 10^{12} \text{ electrons}$$

Total number of electrons emitted in 2 sec = 2 ($N_1 + N_2$)

2. Photoelectric Effect in Classical Domain

Input power: *P*
Fraction of input power radiated: *f*
Work function of the metal: W_0
Diameter of the atom: *D*
Distance of atom from the bulb: *d*

∴ time required for emission

$$\ddot{A}t = \frac{W_0}{fP} \frac{4pd^2}{p\left(D/2\right)^2} = 57.6 \text{ sec}$$

3. Bohr Theory

In transition from n = 3 to n = 1, there are three possible wavelengths that can be emitted:

$\lambda_{3\to1}$ = 6577 A°, $\lambda_{3\to2}$ = 1028 A° and $\lambda_{2\to1}$ = 1219 A°

Nuclear Physics

1. Radioactive Dating

$N_0 (U^{238}) = N_0 (U^{235})$ but $N (U^{238}) : N (U^{235})$ = 140 : 1

$T_{1/2}(U^{238}) = 4.5\times10^9$ years and $T_{1/2}(U^{235}) = 7.13\times10^8$ years

$T_{1/2} = 0.693/\lambda$

$$N = N_0 e^{-l\,t} \Rightarrow t = \frac{log_e\left(N_1/N_2\right)}{\left(l_2 - l_1\right)} = 6.04 \times 10^9 \text{years}$$

2. Medical Application of Radioactivity

Activity $A = dN/dt$, $A = A_0\, e^{-\lambda t}$ and $T_{1/2} = 0.693/\lambda$

$A_0 = 3.7\times10^4$dps and $T_{1/2}$ = 15 hrs

Let *V* be the total volume of blood and

v = 1 *cc* : sample

After *t* = 5 hrs, *a* = 296 dpm in volume *v* whereas $A = 2.937\times10^3$dps

∴ $V = v(A/a)$ = 5953 *cc* = 5.953 litre

3. Nuclear Fusion in the Sun

(i) Release of energy per process = mass excess = 25.7066 MeV

(ii) Total energy ÷ energy per process = no. of processes per sec = 9.725×10^{37} /sec

Mass converted into Helium = no. of processes × m($_2He^4$) = 6.5×10^{11} kg/sec

(iii) Time to convert all mass into Helium at this rate = mass of the Sun ÷ rate of He production = 1.886×10^{11} years

(iv) 1.886×10^{11} years – 5×10^9 years = 1.836×10^{11} years

4. Nuclear Fission in a Nuclear Reactor

Energy per reaction = mass excess = 23.8464 MeV

200 MW at 25% efficiency means the total energy produced must be 800 MW

This requires 800 MW ÷ 23.8464 MeV
= 2.0968×10^{20} reaction

Number of deuterium atoms required per second = $2 \times 2.0968 \times 10^{20} = 4.194 \times 10^{20}$

This translates to 1.402×10^{-6} kg per second or 0.1211 kg per day.

References

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