

# RUBRICS TO REFINE TEACHING SKILLS IN MATHEMATICS AT THE SECONDARY STAGE

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Assessment for learning is the process of seeking and interpreting evidence for use by learners and their teachers to decide where the learners are in their learning, where they need to go and how best to get there. Assessment in education refers to any of the many ways in which someone seeks to gain evidence to help them to measure or to make judgements about the quality, quantity or level of a pupil's learning. The emphasis in assessment for learning is on the active engagement of pupils with assessment as an integral component of the learning process. When we think of the learner as an active participant in constructing his or her own conceptualisation of mathematics, we need to come up with an assessment strategy through which we could assess the progress of learner in different dimensions of the concepts. Rubrics divide an assignment into its component parts and provide a detailed description of what constitutes acceptable or unacceptable levels of performance for each of those parts. Keeping copies of individual learners' rubrics can allow us to pinpoint a learner's continuing improvement or weaknesses over time, rubrics showing learner development over time can also allow us to gain a clearer view of teaching blind spots, omissions, and strengths.

In this paper, a detail about different parts of rubrics has been described. Sufficient number of mathematical illustration has also been provided. It can be a utility tool for teachers to refine their teaching skills in respect of their learners.

## Introduction

In the focus paper of Teaching of Mathematics, NCERT (2005), it has mentioned that with Mathematics students learn to represent quantitative and spatial relationships in a broad range of situations. They can express these relations by using terms, signs, and symbols of mathematics. They learn to use procedures with the signs and symbols, follow rules, carry out numerical and symbolic calculations and make predictions and interpret results based on the use of the procedures. They come to understand the importance for and use of mathematical terms, signs, symbols and process for solving

interdisciplinary problems. The sequence of contextual activities permits students to gradually develop methods for symbolising problem situations. It is also mentioned that instruction in classrooms should not begin by presenting students with the formal terms, signs, symbols, and rules, along with the expectation that students then will appropriately use these formal ideas to solve problems. It has been recommended that classroom leads students to value and understand the need for mathematics through the gradual development of their ability to mathematically represent complex problems, a process called "progressive formalisation."

The textbooks developed on the basis of these recommendations include tasks and questions designed to engage students in mathematical thinking and discourse. Students explore mathematical relationships, develop their own strategies for solving problems, use appropriate problem-solving tools, work together cooperatively, and value one another's strategies. Students are encouraged to explain their thinking as well as their solutions. Teachers are expected to help students develop common understanding and usage of the terms, signs, symbols, and rules of mathematics, which the students then attempt to use in articulating their thinking. Other activities have been designed so that students can extend their ideas to new problem situations.

The monitoring of students' progress has always been a key aspect of the job of teaching. Mathematics teachers traditionally have monitored progress by giving quizzes and chapter tests, scoring and counting the number of correct answers on each, and periodically summarising student performance in terms of a letter grade. The need to consider alternative ways of assessing students' classroom performance grew as a consequence of teachers' role as recommended by NCF-2005.

### **Continuous and Comprehensive Evaluation (CCE)**

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We need to emphasis on the word "Educational Evaluation" first. What we understand that evaluation is an activity which is conducted by the educators/ teachers in order to continuously and systematically review and then enhance the teaching learning process that they are endeavouring to facilitate. Educational evaluation

deals with student's evaluation which includes the assessment of the performance of the students in the areas of their personality development in terms of intellectual, social and emotional development after they have been provided learning experiences through classroom practices.

Evaluation is often confused with the term measurement and both the terms are used synonymously. But both are not the same. The term measurement stands for measuring the performance of the student on a particular scale. The pattern of measurement which is mostly followed in our assessment system relates to marking on a scale of 0-100 marks. This also includes pass-fail system where in all those who secure 33 per cent marks and above are declared pass and below this are tagged fail. This scale is a matter for classifying the students on the basis of the marks they obtain in a test or examination. Therefore, measurement provides a quantitative description of pupils' performance based on artificial classification. It does not include value judgment and thus, it gives a fragmented picture of student's performance. Moreover, all these aspects are related only to intellectual growth.

On the other hand evaluation is broader term as compared to measurement and it includes both quantitative and qualitative description of the performance and value judgment. Regarding quantitative description as written above, measurement on a scale is applied and marks are allotted. For qualitative description interpretation of the marks secured by the student are made in reference to him/herself, her/his group and certain criteria. It also includes value judgment regarding the desirability of

behaviour related to all the domains of personality development.

Evaluation is an integral part of any teaching and learning programme. Whenever a question is asked in a class and answered by a student and the answer is judged by the teacher, evaluation takes place. Thus, both teaching and evaluation go hand in hand with each other. In fact, it is not possible to have teaching and learning without evaluation.

Both teaching and evaluation are based on the instructional objectives which provide direction to them. Instructional objectives are those desirable behaviour which are to be developed in students through the learning experiences. These are reflected in the form of syllabus, instructional material and information given by the teacher. Instructions are given for achieving the objectives and evaluation is done to see whether the instructional objectives have been achieved and to what extent. The interrelationship of objectives, instructional process or the learning experiences and evaluation in a programme of teaching can be expressed more clearly as shown in Fig. 1

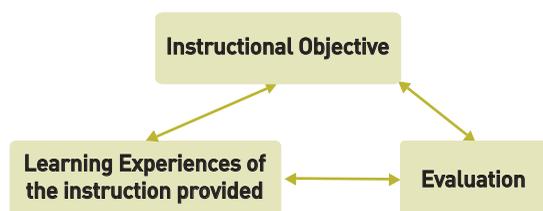


Fig. 1

Fig. 1 illustrates that the three components teaching, learning and evaluation constitute an integrated network in which each components depends on the other. Thus, through evaluation, the teacher not only assesses as to how far the

student has achieved the objectives but also examines the effectiveness of the teaching strategy such as methodologies, means and the materials used for achieving those objectives.

The first step in this change process involves to motivate teachers to see the need to change their assessment practices. This usually comes as a consequence of changing their instructional practices and seeing the quality of the work their students are doing. For this purpose, rubrics may be introduced for the process evaluation of concepts and it may be an effective tool towards continuous and comprehensive evaluation.

## Introducing Rubrics

At its most basic, a rubric is a scoring tool that lays out the specific expectations for an assignment. Rubrics divide an assignment into its component parts and provide a detailed description of what constitutes acceptable or unacceptable levels of performance for each of those parts. Rubrics can be used for grading a large variety of assignments and tasks: Project evaluation, discussion participation, laboratory reports, portfolios, group work, oral presentations, and more.

### Do we need a rubric?

How do we know that we need a rubric? One sure sign is if we check off more than three items from the following list:

- We are getting pain from writing the same comments on almost every student paper.
- We're far behind in our grading.
- Students often complain that they cannot read the notes we labored so long to produce.

- We have graded all our papers and worry that the last ones were graded slightly differently from the first ones.
- We want students to complete a complex assignment that integrates all the work over the term and are not sure how to communicate all the varied expectations easily and clearly.
- We want students to develop the ability to reflect on ill-structured problems but we aren't sure how to clearly communicate that to them.
- We give a carefully planned assignment that we never used before and to our surprise, it takes the whole class period to explain it to students.
- We give a long narrative description of the assignment in the syllabus, but the students continually ask two to three questions per class about our expectations.
- We have worked very hard to explain the complex end-of-term paper; yet students are starting to regard us as incomprehensible assignments.

Rubrics set you on the path to addressing these concerns.

### What are the parts of a rubric?

Rubrics are composed of four basic parts in which the teacher sets out the parameters of the assignment. The parties and processes involved in making a rubric can and should vary tremendously, but the basic format remains the same. In its simplest form, the rubric includes a task description (the assignment), a scale of some sort (levels of achievement, possibly in the form of grades), the dimensions of the assignment (a breakdown of the skills/knowledge involved in the assignment), and descriptions of what constitutes each level of performance (specific feedback) all set out on a grid, as shown in Fig. 2.

	Scale level 1	Scale level 2	Scale level 3
Dimension 1			
Dimension 2			
Dimension 3			
Dimension 4			

Fig 2 : Basic rubric grid format

- We work with our colleagues and collaborate on designing the same assignments for program courses, yet we wonder if our grading scales are different.
  - We've sometimes been disappointed by whole assignments because all or most of the class turned out to be unaware of academic expectations.
- This is the most common, but sometimes we may use more. Rarely, however, we may go over our maximum of five scale levels and six to seven dimensions. We look at the four component parts of the rubric and, using an assignment as an example, provide the above grid *part-by-part* until it is a useful grading tool (a usable rubric) for the teacher and a clear indication of expectations and actual performance for the student.

**Part-by-Part Development of a Rubric**

**Part 1: Task Description**

The task description is almost always originally framed by the teacher and involves a “performance” of some sort by the student. The task can take the form of a specific assignment, such as a paper, a poster, or a presentation.

difficult it becomes to differentiate between them and to articulate precisely why one student’s work falls into the scale level it does. On the other hand, more specific levels make the task clearer for the student and they reduce the teacher’s time needed to furnish detailed grading notes. We have the following grid (Fig. 3) for scaling in rubrics in Mathematics at the secondary stage.

Dimensions	Exemplary	Competent	Developing
Knowledge/Understanding			
Geometrical Skills			
Analytical Skills			
Applications			

**Fig.3** : Part 2: Scales

**Part 2: Scale**

The scale describes how well or poorly any given task has been performed and occupies yet another side of the grid to complete the rubric’s evaluative goal. Terms used to describe the level of performance should be tactful but clear. Here are some commonly used labels compiled:

- Sophisticated, competent, partly competent, not yet competent
- Exemplary, proficient, marginal, unacceptable
- Advanced, intermediate high, intermediate, novice
- Distinguished, proficient, intermediate, novice
- Accomplished, average, developing, beginning.

There is no set formula for the number of levels a rubric scale should have. We should prefer to clearly describe the performances at three levels using a scale. The more levels there are, the more

**Part 3: Dimensions**

The dimensions of a rubric lay out the parts of the task simply and completely. A rubric can also clarify for students how their task can be broken down into components and which of those components are most important. Is it calculation? The analysis? The factual content? The process techniques? And how much weight is given to each of these aspects of the assignment? Although it is not necessary to weight the different dimensions differently, adding points or percentages to each dimension further emphasises the relative importance of each aspect of the task. Dimensions should actually represent the type of component skills students must combine in a successful scholarly work, such as the need for a firm grasp of content, technique, citation, examples, analysis, and a use of language appropriate to the occasion. When well done, the dimensions of a rubric (usually listed along one

side of the rubric- Fig. 3) will not only outline these component skills, but after the work is graded, should provide a quick overview of the student's strengths and weaknesses in each dimension.

Breaking up the assignment into its distinct dimensions leads to a kind of task analysis with the components of the task clearly identified. Both students and teachers find this useful. It tells the student much more than a mere task assignment or a grade reflecting only the finished product. Together with good descriptions, the dimensions of a rubric provide detailed feedback on specific parts of the assignment and how well or poorly those were carried out.

#### Part 4: Description of the Dimensions

Dimensions alone are all-encompassing categories, so for each of the dimensions, a rubric should also contain at the very least a description of the highest level of performance in that dimension. A rubric that contains only the description of the highest level of performance is called a scoring guide rubric. Scoring guide rubrics allow for greater flexibility and the

personal touch, but the need to explain in writing where the student has failed to meet the highest levels of performance does increase the time it takes to grade using scoring guide rubrics.

As per the secondary stage mathematics syllabus, we have selected 7 themes:

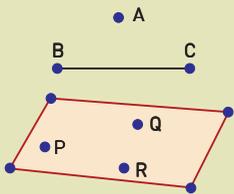
1. Geometry
2. Trigonometry
3. Number system
4. Algebra
5. Statistics
6. Coordinate Geometry
7. Mensuration

In this article, as per the syllabus of Secondary Mathematics we have listed tasks for the theme Geometry and rubrics for the several task in the following:

- **Basic Concepts-1**

**1. Task:** To check whether the student has understood the basic terminology (Point, Line, Plane etc.) of Geometry

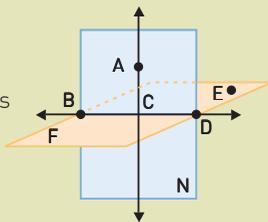
**Dimensions** — Knowledge/Understanding

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>• The student understands the meaning of the word Geometry. S/he is able to recognise and draw the basic building blocks of geometry viz., point, line and plane and knows that they are undefined terms.</li> <li>• S/he is able to recognise the geometrical shapes in his surroundings.</li> <li>• S/he is also able to define, recognise and draw terms like collinear points, coplanar points, line</li> </ul> 	<ul style="list-style-type: none"> <li>• The student understands the meaning of the word Geometry. S/he does not know that the basic building blocks of Geometry viz., point, line and plane are undefined. S/he is able to draw basic building blocks.</li> <li>• S/he is able to recognise the geometrical shapes in his surroundings.</li> </ul>	<ul style="list-style-type: none"> <li>• The student understands the meaning of the word Geometry.</li> <li>• S/he is not able to recognise the geometrical shapes in his surroundings.</li> <li>• S/he is able to draw points, lines etc.</li> </ul>

<p>segment , ray , intersecting lines , parallel lines ,concurrent lines etc.</p> <p>S/he is able to solve the following question successfully.</p> <p>Q.1. Draw and label each of the following:</p> <p>(a) a segment with endpoints U and V</p> <p>(b) opposite rays with a common end point Q</p>	<ul style="list-style-type: none"> <li>S/he is not able to define terms like collinear points, coplanar points, line segment, ray, intersecting lines, parallel lines, concurrent lines etc. but he is able to draw them.</li> <li>S/he is able to solve the question.</li> </ul>	<p>but not able to draw intersecting lines, parallel lines, concurrent lines etc.</p> <ul style="list-style-type: none"> <li>S/he is able to solve the question but not able to label the figures correctly.</li> </ul>
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**Dimensions — Analytical Skill**

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>The student applies her/his problem solving skills to solve the following question successfully :</li> </ul> <p>Q. Use the figure to name each of the following :</p> <p>(a) Three points</p> <p>(b) Two lines</p> <p>(c) Two planes</p> <p>(d) One ray</p> <p>(e) Intersecting lines</p> <p>The student understands the meaning of the following statement :</p> <p>If two rays share a common end point , then they form a line.</p>	<ul style="list-style-type: none"> <li>The student is able to recognise and name points and lines but unable to recognise plane, ray and intersecting lines.</li> </ul>	<ul style="list-style-type: none"> <li>The student is able to recognise points and line but not able to name them.</li> </ul>



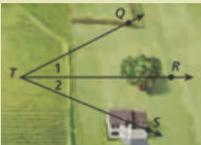
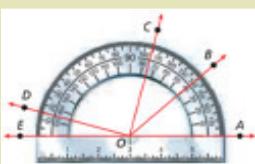
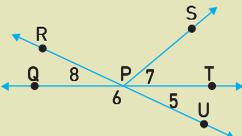
**Dimensions — Application**

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>The student is able to recognise a sheet of paper as a plane.</li> <li>S/he is able to draw a line segment by paper folding.</li> <li>S/he is also able to locate a point as the intersection of two line segments by paper folding.</li> </ul>	<ul style="list-style-type: none"> <li>The student is able to recognise a sheet of paper as a plane. He is not able to draw line segment and point using paper folding.</li> </ul>	<ul style="list-style-type: none"> <li>The student is not able to understand paper folding activities.</li> </ul>

• **Basic Concepts- 2**

**1. Task:** To check whether the student has understood the concepts of angles and its types.

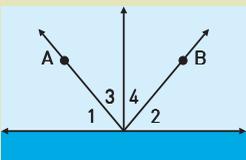
**Dimensions — Knowledge/Understanding**

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>The student understands the definition, naming conventions, interior and exterior of an angle.</li> </ul> <p>Student is able to answer the following question correctly.</p> <p>Q.1 A surveyor recorded the angles formed at a point T and three distant points Q, R and S.</p>  <p>Name the three angles</p> <ul style="list-style-type: none"> <li>The student knows how to measure an angle in degrees using a protactor.</li> </ul> <p>The student is able to answer the following question correctly.</p> <p>Q.2. Find the measure of angle formed by rays OA and OB and OA and OE.</p>  <ul style="list-style-type: none"> <li>Student is able to add two given angles.</li> <li>The student is able to define, recognise and draw acute angles, right angle, obtuse angle, straight angle, reflex angle, complete angle and zero angle.</li> </ul> <p>Student is able to solve the following question successfully</p> <p>Q.3. Identify right angle , acute angle , obtuse angle and straight angle in the following Figures :</p>  <ul style="list-style-type: none"> <li>The student is able to define, recognise and draw various pairs of angles viz; adjacent angles, complementary angles, supplementary angles, vertically opposite angles and linear pair.</li> </ul> <p>The student is able to solve the following question successfully :</p> <p>Q.4. Identify adjacent angles, vertically opposite angle and linear pair of angles in the following Figure :</p> 	<ul style="list-style-type: none"> <li>The student is able to define an angle but he is not able to name angles. He is able to identify interior and exterior of an angle. Student is able to solve Q.1. partially. He is able to identify the angles but not able to name them.</li> <li>The student is able to measure angle in degree using protactor. Student is able to measure angle AOB but unable to measure angle AOE in Q.2.</li> <li>The student is able to add angles.</li> <li>The student is able to define and recognise different types of angles. S/he is not able to draw them. S/he is able to solve Q.3 successfully.</li> <li>The student is able to define and recognise various pairs of angles. He is not able to draw them. Student is able to solve Q.4 successfully.</li> </ul>	<ul style="list-style-type: none"> <li>The student is able to identify the interior and exterior of an angle. He is not able to define and name angles. S/he is not able to solve 1.</li> <li>The student is not able to measure angles using protector. He is not able to solve Q.2.</li> <li>The student is able to add angles.</li> <li>The student is able to define different types of angles. He is not able to draw and recognise them. He is not able to solve Q.3.</li> <li>The student is able to define various pairs of angles. He is not able to draw and recognise them. He is not able to solve Q.4.</li> </ul>

Dimensions — Analytical Skill

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>The student has the skill to solve the following question successfully.</li> </ul> <p>Q.1. Given "A = 56° and "B = (2x - 4)°, find the measure of each of the following.</p> <p>(a) Supplement of angle A                      (b) Complement of angle A                      (c) Supplement of angle B                      (d) Complement of angle B</p>	<ul style="list-style-type: none"> <li>The student is able to understand Q.1 but he is not able to apply concepts of supplementary angles correctly.</li> </ul>	<ul style="list-style-type: none"> <li>The student understands that measure of two angles are given but he is not able to apply concepts of complementary and supplementary angles.</li> </ul>

Dimensions — Application

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>The student is able to apply the knowledge of angles to solve the following question successfully.</li> </ul> <p>Q.1. A sprinkler swings back and forth between A and B in such a way that <math>\angle 1 = \angle 2</math>. <math>\angle 1</math> and <math>\angle 3</math> are complementary and <math>\angle 2</math> and <math>\angle 4</math> are complementary. If <math>\angle 1 = 45^\circ</math>, find <math>\angle 2</math>, <math>\angle 3</math> and <math>\angle 4</math>.</p> 	<ul style="list-style-type: none"> <li>The student is not able to apply concept of complementary angles.</li> </ul>	<ul style="list-style-type: none"> <li>The student is not able to understand the question.</li> </ul>

Similarly, the reader may develop rubrics for the following task related to Basic concepts of Geometry theme.

• **Basic Concepts- 3**

**3. Task:** To check whether the student has understood what are curves, polygons, triangles,

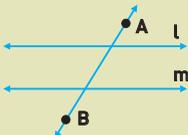
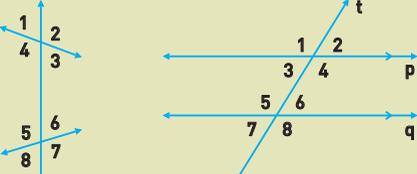
quadrilaterals and circles.

• **Lines and Angles - 1**

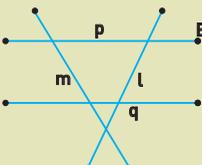
**4. Task:** To check whether the student has understood the concept of a transversal and angles formed by a transversal when it intersects two or more parallel or non-parallel lines.

Dimensions — Knowledge/Understanding

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>The student understands the meaning of a transversal. He can define, identify and draw a transversal.</li> </ul>	<ul style="list-style-type: none"> <li>The student is able to identify a transversal only when it intersects two parallel lines. He is able to</li> </ul>	<ul style="list-style-type: none"> <li>The student is not able to define a transversal. He is unable to solve Q.1.</li> </ul>

<p>The student is able to solve the following questions successfully :</p> <p>Q.1. Identify transversals in the following figures.</p>  <ul style="list-style-type: none"> <li>S/he is able to identify the interior and exterior region when transversal intersects parallel/ non-parallel lines.</li> </ul> <p>The student is able to solve the following question successfully :</p> <p>Q.2. Identify the interior and exterior region in the following figure</p>  <ul style="list-style-type: none"> <li>The student is able to identify , name and draw angles formed when a transversal intersects parallel / non-parallel lines.</li> </ul> <p>Student is able to solve the following question successfully :</p> <p>Q.3. Identify and name pairs of corresponding, interior alternate angles, exterior alternate angles, co-interior angles in the following figures :</p> 	<p>identify line AB of figure (ii) of Q.1. as transversal.</p> <ul style="list-style-type: none"> <li>The student is able to identify interior and exterior region when two parallel lines is intersected by a transversal. Student is able to solve Q.2.</li> <li>The student is able to identify , name and draw angles formed when a transversal intersects parallel lines. He is able to solve part (ii) of Q.3.</li> </ul>	<ul style="list-style-type: none"> <li>The student is able to identify exterior region only. Student is able to solve Q.2 partially.</li> <li>The student is not able to solve Q.3.</li> </ul>
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Dimensions — Analytical Skill

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>The student is able to solve the following question : Identify and mark pairs of corresponding, alternate and co-interior angles in the following Figure</li> </ul> 	<ul style="list-style-type: none"> <li>The student is able to solve Q.1. partially. He understands that there can be only one transversal for one set of parallel lines.</li> </ul>	<ul style="list-style-type: none"> <li>The student is not able to solve Q.1.</li> </ul>

Similarly, the reader may develop rubrics for the following task related to lines and angles of Geometry theme.

• Lines and Angles - 2

**5. Task:** To check whether the student has understood the concepts of relationship between two corresponding angles, two alternate angles and two co-interior angles when a transversal intersects a pair of parallel lines.

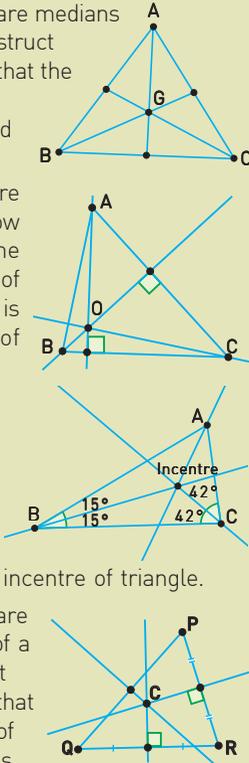
• Triangle - 1

**6. Task:** To check whether the student has understood the concept of angle sum property of a triangle.

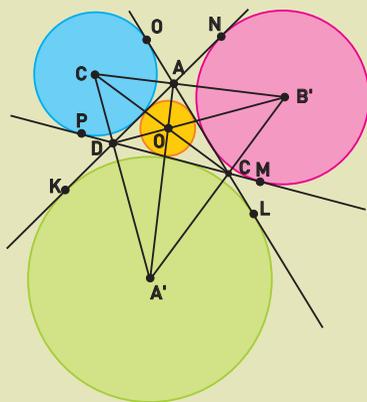
• Triangle - 2

**7. Task:** To check whether the student has understood the concepts of median, altitude, angle bisector, perpendicular bisector and points of their concurrency.

Dimensions — Knowledge/Understanding

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>The student knows what are medians of a triangle, how to construct them and he also knows that the point of concurrency of medians is called centroid (G) of triangle.</li> <li>The student knows what are altitudes of a triangle, how to construct them and he also knows that the point of concurrency of altitudes is called orthocenter (O) of triangle.</li> <li>The student knows what are angle bisectors of a triangle, how to construct them and he also knows that the point of concurrency of angle bisectors is called incentre of triangle.</li> <li>The student knows what are perpendicular bisectors of a triangle, how to construct them and he also knows that the point of concurrency of perpendicular bisectors is called circumcentre (C) of triangle.</li> </ul> 	<ul style="list-style-type: none"> <li>The student knows what are medians of a triangle. He knows how to construct them and s/he also knows that the point of concurrency of medians is called centroid of triangle.</li> <li>The student knows what are altitudes of a triangle. He does not know how to construct them. He knows that the point of concurrency of altitudes is called orthocentre of triangle.</li> <li>The student knows what are angle bisectors of a triangle. s/he does not know how to construct them. He knows that the point of concurrency of angle bisectors is called incentre of triangle.</li> <li>The student knows what are perpendicular bisectors of a triangle. S/he does not know how to construct them. He knows that the</li> </ul>	<ul style="list-style-type: none"> <li>The student knows what are medians of a triangle. S/he is not able to construct them and he does not know that the point of concurrency of medians is called centroid of triangle.</li> <li>The student knows what are altitudes of a triangle. S/he is not able to construct them and he does not know that the point of concurrency of altitudes is called orthocentre of triangle.</li> <li>The student knows what are angle bisectors of a triangle. S/he is not able to construct them and he does not know that the point of concurrency of angle bisectors is</li> </ul>

- The student knows what are angle bisectors a triangle, how to construct them and the point, where angle bisector of one of the internal angles of a triangle meets with the angle bisectors of the other two exterior angles of the same triangle, is called the ex-centre of the triangle.



point of concurrency of perpendicular bisectors is called circumcentre of triangle.

- The student knows what are angle bisectors. S/he is not able to construct them. S/he is able to define ex-centre of a triangle.

called incentre of triangle.

- The student knows what are perpendicular bisectors of a triangle. S/he is not able to construct them and he does not know that the point of concurrency of perpendicular bisector is called circumcentre of triangle.
- The student knows what are angle bisectors. S/he is not able to construct them. He is not able to define ex-centre.

### Dimensions — Analytical Skill

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>The student understands the property that centroid divides the median in the ratio 2:1 from vertex side.</li> <li>The student knows that the centroid always lies inside the triangle irrespective of the type of triangle.</li> <li>The student knows the position of incentre , orthocenter , ex-centre and circumcentre in different types of triangles.</li> </ul>	<ul style="list-style-type: none"> <li>The student understands the property that centroid divides the median in the ratio 2:1 from vertex side.</li> <li>The student does not know the position of centroid in different types of triangles.</li> <li>The student does not knows the position of incentre, orthocenter, ex-centre and circumcentre in different types of triangles.</li> </ul>	<ul style="list-style-type: none"> <li>The student does not know about the ratio in which it divides the median. He is also unaware of the position of the centroid in different types of triangles.</li> <li>The student does not knows the position of incentre , orthocenter , ex-centre and circumcentre in different types of triangles.</li> </ul>

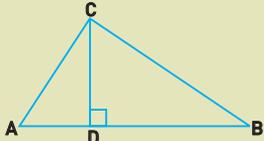
Similarly, reader may develop rubrics for the following tasks related to Triangle of Geometry theme.

- Triangle- 3
- 8. Task: To check whether the student has understood the inequalities in triangle
- Triangle - 4
- 9. Task: To check whether the student has understood the meaning of congruence and different criteria of congruence viz SSS , SAS , ASA , AAS and RHS.
- Triangle- 5

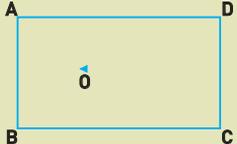
10. Task: To check whether the student is able to understand the concepts of Isosceles and Equilateral Triangles.

- Triangle- 6
- 11. Task: To check whether the student is able to understand the concept of similar triangles.
- Triangle- 7
- 12. Task: To check whether the student is able to understand the Pythagoras Theorem and its applications.

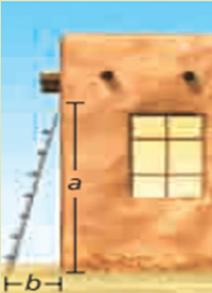
Dimensions — Knowledge/Understanding

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>• The student understands the statement of Pythagoras Theorem and he is also able to prove it.</li> <li>• The student understands the converse of Pythagoras theorem and s/he is also able to prove it.</li> </ul> <p>Student is able to solve the following question correctly :</p> <p>Q.1. In Figure, "<math>\angle ACB = 90^\circ</math> and <math>CD \perp AB</math>.</p> <p>Prove that <math>\frac{BC^2}{AC^2} = \frac{BD}{AD}</math></p> 	<ul style="list-style-type: none"> <li>• The student knows the statement of Pythagoras Theorem but s/he is not able to prove it.</li> <li>• The student knows the statement of converse of Pythagoras Theorem but s/he is not able to prove it.</li> <li>• The student is able to recognise the triangles where s/he can apply the Pythagoras Theorem, but s/he is not able to apply it symbolically.</li> </ul>	<ul style="list-style-type: none"> <li>• The student knows the statement of Pythagoras Theorem but s/he is not able to prove it.</li> <li>• The student is not able to understand the converse of Pythagoras Theorem.</li> <li>• The student is not able to solve the question.</li> </ul>

Dimensions — Analytical Skill

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>• The student has the skills to solve the following question correctly.</li> </ul> <p>O is any point inside a rectangle ABCD (see Figure below). Prove that <math>OB^2 + OD^2 = OA^2 + OC^2</math></p> 	<ul style="list-style-type: none"> <li>• The student is able to make suitable construction but he is not able to apply the concept of Pythagoras Theorem to solve the question.</li> </ul>	<ul style="list-style-type: none"> <li>• The student is not able to understand that the question can be solved by using Pythagoras Theorem.</li> </ul>

Dimensions — Application

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>The student is able to apply his knowledge of Pythagoras Theorem to find the solution of following question : To prevent a ladder from shifting, safety experts recommend that the ratio of a : b be 4 : 1. How far from the base of the wall should you place the foot of a 5 meter ladder? Round to the nearest cm. Refer the following Figure.</li> </ul> 	<ul style="list-style-type: none"> <li>The student understands that he has to apply Pythagoras Theorem to solve the problem, but he is not able to understand the ratio of lengths.</li> </ul>	<ul style="list-style-type: none"> <li>The student is not able to understand the question.</li> </ul>

Readers are encouraged to develop rubrics of listed task on their own also.

• **Quadrilateral- 1**

**13. Task:** To check whether the student has understood the concept of angle sum property of a quadrilateral.

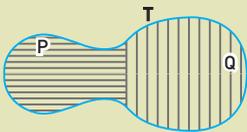
• **Quadrilateral- 2**

**14. Task:** To check whether the student has understood various properties of a quadrilateral and mid point theorem.

• **Quadrilateral- 3**

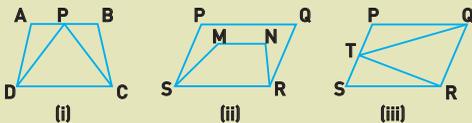
**15. Task:** To check whether the student has understood the concept of area of congruent figures and area of triangle and parallelogram on same base and between same parallel lines.

Dimensions — Knowledge/Understanding

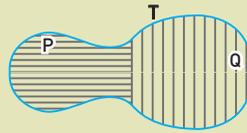
Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Understands that if two Figures are congruent then they have equal area.</li> <li>Understands that if a planar region formed by a Figure T is made up of two non-overlapping planar regions formed by figures P and Q, then <math>ar(T) = ar(P) + ar(Q)</math>.</li> <li>Understands that two Figures are said to be on the same base and between the same parallels, if</li> </ul> 	<ul style="list-style-type: none"> <li>Understands that if two figures are congruent then they have equal area. Has misunderstanding that if two figures have equal area then they are congruent.</li> <li>Understands that if a planar region formed by a figure T is made up of two non-overlapping planar regions formed by Figures P and Q, then <math>ar(T) = ar(P) + ar(Q)</math>.</li> </ul>	<ul style="list-style-type: none"> <li>Unable to understand the relationship of congruent figures with area.</li> <li>Unable to understand the meaning of non-overlapping regions.</li> <li>Unable to understand that two Figures are said to</li> </ul>

they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base. Able to solve the following question correctly

Q.1. Which of the following Figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.



- Understands and prove that parallelograms on the same base and between the same parallels are equal in area.
- Understands and prove that parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels.
- Understands and prove that two triangles on the same base (or equal bases) and between the same parallels are equal in area.
- Understands that two triangles having the same base (or equal bases) and equal areas lie between the same parallels.
- Understands that if a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram
- Understands and prove that a median of a triangle divides it into two congruent triangles.



- Not able to understand the meaning of vertices (or vertex) opposite to the common base.

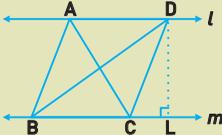
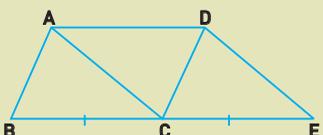
Able to give correct answer to part (i) of Q.1.

- Understands that parallelograms on the same base and between the same parallels are equal in area.
- Understands that parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels.
- Understands that two triangles on the same base (or equal bases) and between the same parallels are equal in area.
- Understands that two triangles having the same base (or equal bases) and equal areas lie between the same parallels.
- Understands that if a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram
- Understands that a median of a triangle divides it into two congruent triangles.

be on the same base and between the same parallels, if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each Figure lie on a line parallel to the base.

- Unable to understand the relationship between areas of triangle and parallelogram on same or equal bases and between same parallels.

Dimensions — Analytical Skill

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Able to solve the following questions successfully:</li> </ul> <p>Q.1. In Fig., the area of <math>\Delta ABC</math> is given to be <math>18 \text{ cm}^2</math>. If the altitude <math>DL</math> equals <math>4.5 \text{ cm}</math>, find the base <math>BC</math> of the <math>\Delta BCD</math>.</p>  <p>Q.2. In Fig., <math>ABCD</math> and <math>ACED</math> are two parallelograms. If area of <math>\Delta ABC</math> equals <math>12 \text{ cm}^2</math>, and the length of <math>CE</math> and <math>BC</math> are equal, find the area of the trapezium <math>ABED</math>.</p> 	<ul style="list-style-type: none"> <li>Understands that Area of <math>DABC</math> and area of <math>DBCD</math> are equal but not able to apply formula for area of triangle. [Q.1.]</li> <li>Understands that area of trapezium is equal to area of parallelogram <math>ABCD</math> plus area of triangle <math>DCE</math>, but unable to solve the questions.</li> </ul>	<ul style="list-style-type: none"> <li>Unable to solve the question (Q.1.)</li> <li>Unable to solve the question (Q.2.)</li> </ul>

Dimensions — Application

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Applies knowledge to find the solution of following problem successfully.</li> </ul> <p>Q.1. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.</p>	<ul style="list-style-type: none"> <li>Understands that question requires the application of concept that area of triangle on same base and between same parallels are same. Unable to draw Figure.</li> </ul>	<ul style="list-style-type: none"> <li>Unable to solve the question.</li> </ul>

• Circle – 1

**16. Task:** To check whether the student has understood the meaning of a circle and its parts.

• Circle – 2

**17. Task:** To check whether the student has understood the results relating to congruent arc,

equal chords and angle subtended by them at the centre.

• Circle – 3

**18. Task:** To check whether the student has understood the results relating to perpendicular from the centre of a circle to its chord, and the chords of a circle equidistant from the centre.

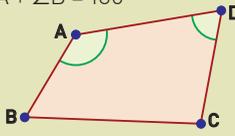
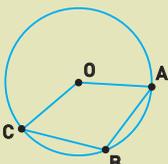
• Circle – 4

**19. Task:** To check whether the student has understood the results relating to angle subtended by an arc of a circle at the centre and at the remaining part of the circle.

• Circle – 5

**20. Task:** To check whether the student has understood the meaning of a cyclic quadrilateral and its properties.

Dimensions — Knowledge/Understanding

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Understands the meaning of a cyclic quadrilateral.</li> <li>Identifies cyclic quadrilateral from a given collection of figures.</li> <li>States the following results :                             <ul style="list-style-type: none"> <li>(i) The sum of either pair of opposite angles of a cyclic quadrilateral is <math>180^\circ</math></li> <li>(ii) If the sum of a pair of opposite angles of a quadrilateral is <math>180^\circ</math>, then the quadrilateral is cyclic.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>Understands the meaning of cyclic quadrilateral and identifies cyclic quadrilateral from a given collection of quadrilaterals.</li> <li>States the result (i) correctly but does not state (ii) properly.</li> </ul> <p>For example, states that quadrilateral ABCD is cyclic if <math>\angle A + \angle B = 180^\circ</math></p> 	<ul style="list-style-type: none"> <li>Understands the meaning of a cyclic quadrilateral but also describes a quadrilateral given below as cyclic.</li> </ul>  <p>States the result (i) correctly but does not state the result (ii)</p>

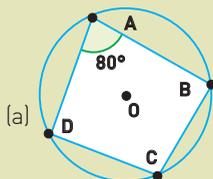
Dimensions — Analytical Skill

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Proves the result (i) and (ii) with proper reasoning.</li> </ul>	<ul style="list-style-type: none"> <li>Verifies the results (i) and (ii) through activities.</li> </ul>	<ul style="list-style-type: none"> <li>Verifies the result (i) through an activity.</li> </ul>

Dimensions — Application

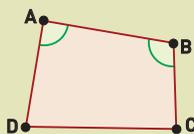
Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Applies the results (i) to (iii) in solving geometrical problems with reasoning and proper Figure if necessary.</li> </ul>	<ul style="list-style-type: none"> <li>Applies the results (i) and (ii) in solving direct geometrical problems such as in direct geometrical problems such as in (a) and (b)</li> </ul>	<ul style="list-style-type: none"> <li>Applies the result (i) correctly in (a) and finds <math>\angle DCB = 100^\circ</math></li> </ul>

- Applies the results (i) to (iii) in solving geometrical problems with reasoning and proper figure if necessary.



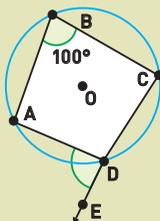
(a) What is  $\angle DCB$  ?

(b) If  $\angle A + \angle C = 170^\circ$

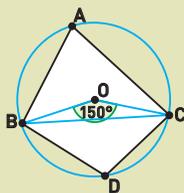


Is ABCD, a cyclic quadrilateral?

(c) In the following figure what is  $\angle ADE$ ?



(d) O is the centre of the circle , what is the measure of  $\angle BDC$  ?



In this question , describes BOCD a cyclic quadrilateral and finds  $\angle BDC = 30^\circ$

(e) Is not able to prove the result : An isosceles trapezium is cyclic.  
At least draws the figure correctly and understands the meaning and properties of isosceles trapezium.

(b) does not decide whether ABCD is cyclic or not.

(c) finds  $\angle ADE = 80^\circ$  by understanding that  $\angle B + \angle ADE = 180^\circ$

(d) finds  $\angle BDC = 30^\circ$  using  $\angle BOC + \angle BDC = 180^\circ$

(e) Does not understand the meaning of an isosceles trapezium and does not draw even the figure.

• Circle – 6

**21. Task:** To check whether the student has understood the meaning of tangent and secant to a circle and their properties.

• Constructions – 1

**22. Task:** To check whether the student has understood basic geometrical constructions such as drawing a line segment (angle) equal to a given line segment (angle), parallel lines, perpendicular lines and triangles with given elements.

Dimensions — Knowledge/Understanding

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Understands the meaning of following constructions :                             <ul style="list-style-type: none"> <li>(i) Line segment equal to given line segment.</li> <li>(ii) An angle equal to given angle.</li> <li>(iii) Perpendicular to a given line through a point on it (outside it).</li> <li>(iv) Bisector of a given line segment.</li> <li>(v) Bisector of an angle.</li> <li>(vi) A line parallel to a given line through a point not lying on the line.</li> <li>(vii) Triangle with three given sides (SSS).</li> <li>(viii) Triangle with two sides and included angle (SAS).</li> <li>(ix) Triangle with two angles and included side (ASA).</li> <li>(ix) Right triangle with hypotenuse and one side (RHS).</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>Understands the meaning of constructions (i) to (x).</li> </ul>	<ul style="list-style-type: none"> <li>Understands the meaning of constructions (i) to (x) except                             <ul style="list-style-type: none"> <li>(a) In (iii), understands perpendicular to a given line through a point on it and not a point outside it.</li> <li>(b) does not understand meaning of included angle and included side in (viii) and (ix) respectively.</li> </ul> </li> </ul>

Dimensions — Analytical Skill

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Does all the constructions (i) to (x) accurately and neatly using ruler and compass.</li> <li>Writes the steps of constructions.</li> <li>Gives justification of each constructions (i) to (x)</li> </ul>	<ul style="list-style-type: none"> <li>Does all the constructions (i) to (x) using a ruler and compass.</li> <li>Writes the steps of construction but skips some steps in between</li> <li>Gives justification for constructions (i) to (vi)</li> </ul>	<ul style="list-style-type: none"> <li>Does all the constructions (i) to (x) using a ruler and compass but in some cases finds convenient to use set squares and protractor specially in drawing parallel lines and in drawing angles in constructing triangles (viii) to (ix)</li> <li>Ignores the word 'included angle' and 'included side' in constructions (viii) and (ix) respectively.</li> <li>Does not give justification of any construction</li> </ul>

**Dimensions — Application**

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Applies the knowledge of basic constructions (mainly bisector of an angle) in constructing angles of measures <math>60^\circ</math>, <math>120^\circ</math>, <math>90^\circ</math>, <math>30^\circ</math>, <math>45^\circ</math>, <math>75^\circ</math>, <math>105^\circ</math>, <math>135^\circ</math>, <math>150^\circ</math>, <math>165^\circ</math>, and <math>195^\circ</math> using a ruler and compass accurately and neatly.</li> </ul>	<ul style="list-style-type: none"> <li>Applies the knowledge of basic constructions (mainly bisector of an angle) in constructing all these angles except angle of <math>195^\circ</math> using a ruler and compass.</li> </ul>	<ul style="list-style-type: none"> <li>Applies the knowledge of basic constructions (mainly bisector of an angle) and draws angles of <math>60^\circ</math>, <math>120^\circ</math>, <math>90^\circ</math>, <math>30^\circ</math>, <math>45^\circ</math>, <math>75^\circ</math>, <math>135^\circ</math> only</li> </ul>

• **Constructions – 2**

**23. Task:** To check whether the student has understood how to construct triangles with base, base angle and sum / difference of two sides and with base angles and perimeter.

• **Construction – 3**

**24. Task:** To check whether the student has understood how to divide a line segment into given number of equal parts and how to divide a line segment in a given ratio.

• **Construction – 4**

**25. Task:** To check whether the student has understood how to construct a triangle similar to a given triangle with a given scale factor.

• **Construction – 5**

**26. Task:** To check whether the student has understood how to draw tangents to a circle from a point outside a circle.

• **Euclid Geometry – 1**

**27. Task:** To check whether the student has understood the need of 'undefined' terms in view of the definition given by Euclid with regard to a point line and a plane.

• **Euclid Geometry – 2**

**28. Task:** To check whether the student has understood the meaning of Euclid's axioms and postulates and their use in solving geometric problems.

**Dimensions — Knowledge/Understanding**

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Understands the meaning of an axiom and postulate as suggested by Euclid.</li> <li>Understands that these days axioms and postulates are used interchangeably.</li> <li>States all the Euclid's postulates and some of the axioms.</li> </ul>	<ul style="list-style-type: none"> <li>Understands the meaning of an axiom and postulate as suggested by Euclid but does not understand that postulate are specific to geometry only.</li> <li>States only Euclid's postulates.</li> </ul>	<ul style="list-style-type: none"> <li>Does not understand the meaning of axioms and postulates.</li> <li>States only Euclid's fifth postulate.</li> </ul>

Dimensions — Analytical Skill

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Analyses Euclid’s postulates and appreciates the present modified versions of it and also appreciates the evolution of non-Euclidean geometry as a result of discussion on Euclid’s fifth postulate.</li> </ul>	<ul style="list-style-type: none"> <li>Analyses first two postulates and appreciates their present modified versions , but not able to understand the modified version of fifth postulate.</li> </ul>	<ul style="list-style-type: none"> <li>Does not find any difference between Euclid’s postulates and present modified version of postulates.</li> </ul>

Dimensions — Application

Exemplary	Competent	Developing
<ul style="list-style-type: none"> <li>Solves geometric problems using Euclid’s axioms and postulates through proper deductive reasoning specifying some extra assumptions.</li> </ul>	<ul style="list-style-type: none"> <li>Solve geometric problems using Euclid’s axioms and postulates but does not give proper reasoning.</li> </ul>	<ul style="list-style-type: none"> <li>Is not able to solve any geometric problems using Euclid’s axioms and postulates.</li> </ul>

**Conclusion**

Therefore, using rubrics during evaluation is sort of diagnostic purposes. It enables in finding out the learning difficulties of a child in a particular dimension with reference to conceptual understanding, process of learning, language deficiency, etc. Using rubrics during testing helps in diagnosing the hard spots of learning as well as the learning problems. The learning problems in mathematics may be in any of the dimensions given above for example, understanding in computing and recognition of symbols where the children generally commits mistake. The teacher is supposed to find out the specific difficulty of

the child in learning a concept or a particular step in solving a problem. Therefore, if diagnosis of hard spots of learning is properly done and suitable remedial measures are taken, the learning attainment as well as learning pace of the weak and low achievers will certainly improve. Therefore, using rubrics may be a very suitable and sustainable tools for comprehensive and continuous evaluation.

In the next part of the article we will explore how the use of rubrics can encourage critical thinking and how rubrics can help us to refine our teaching skills. We will also develop rubrics for next themes trigonometry, number system, etc. in next set of articles.

## References

COLUMBIA UNIVERSITY. Standards-Based Mathematics Assessment in Middle School, Rethinking Classroom Practice. Teachers College. New York and London

*Continuing Professional Development – A Practical Guide for Teachers and Schools*. Second Edition, London and New York in Association with The Open University.

*Geometry – An Investing approach*. Key Curriculum Press, CA 4608

*Introduction to Rubrics – An Assessment Tool to Save Grading Time, Convey Effective Feedback and Promote Student Learning* by Dannelle D. Stevens and Antonia Levi Stylus Publishing, LLC Sterling, Virginia. 2016.

*Learning to Teach Mathematics in the Secondary School*. Routledge, New York. NY 10016

NCERT. *Mathematics* Textbook for Class IX. New Delhi

\_\_\_\_\_. *Mathematics* Textbook for Class X. New Delhi

\_\_\_\_\_. *Teaching of Mathematics*. Focus paper, 2005. New Delhi

*National Curriculum Framework– 2005*. NCERT, New Delhi

*Teaching Secondary Mathematics with ICT*. Open University Press. McGraw-Hill Education, England

The Essentials of Mathematics, Grades 7-12. Effective Curriculum, Instruction and Assessment – Association for Supervision and Curriculum Development. Alexandria, VA 22311-1714 USA