PROBLEM BASED LEARNING IN BASIC PHYSICS - IV

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In this article- fourth in the series of articles we present problems for a problem based learning course from the area of heat, thermodynamics and optics. We present the learning objectives in this area of basic physics and what each problem tries to achieve with its solution.

In this article, fourth in the series of Problem Based Learning in Basic Physics, we present problems on heat, thermodynamics and optics. Methodology and philosophy of selecting these problems are already discussed. (Pradhan 2009, Mody 2011)

To review methodology in brief, we note here that this PBL (Problem Based Learning)starts after students have been introduced to formal structure of Physics. Ideally students would attempt only main problem. If they find it difficult, then depending upon their area of difficulty, right auxiliary problem have to be introduced by teacher who is expected to be a constructivist facilitator. Teacher may choose as per her/his requirement or may construct questions on the spot to guide student to right idea and method.

H: Heat and thermodynamics

Learning Objectives

1. To become familiar with heat as a form of energy (that due to random motion).
2. To understand effect of heat on physical properties.
3. Effect of heat on parameters like pressure, temperature etc.
4. Transfer of heat in different processes especially conduction and radiation.
5. Black body radiation and temperature.
6. To understand mathematical structure dealing with above mentioned points.

Problems

1. A narrow necked vessel contains 6 gm of a gas at 7°C and at a certain pressure. The vessel is heated to 147°C, when some of the gas escapes. The pressure of the gas in the vessel is constant. What mass of the gas has escaped? *

   Tasks involved in this problem are:

   (a) To apply PV = nRT before and after.

   (b) Finding out n in the final state and hence how much gas escapes.
2. An air bubble starts rising from the bottom of a lake, 2.5 metres deep. Its diameter increases from 3.6 mm at the bottom to 4 mm at the surface. If the temperature of water on the surface is 40°C, what is the temperature at the

bottom of the lake? [Atmospheric pressure = 0.76 m of Hg; r_{Hg} = 13600 kg/m^3; r_{water} = 1000 kg/m^3; g = 9.8 m/s^2] [Mistry]

Tasks involved in this problem are:

(a) Use of $PV = nRT$ knowing that 'n' remains constant .

(b) Using $P = P_{atm} + hñg$

3. A closed vessel contains liquid water in equilibrium with its vapour at 100°C and 1 atm. One gram of water vapour at this temperature and pressure occupies a volume of 1670 cm^3. The heat of vaporisation at this temperature is 2250 J/gm. (a) How many molecules are there per cm^3 of vapour. (b) How many vapour molecules strike each cm^2 of liquid surface per second? (c) If each molecule which strikes the surface condenses, how many evaporate from each cm^2 per second? (d) Compare the mean kinetic energy of a vapour molecule with the energy required to transfer one molecule from the liquid to the vapour phase. *

Tasks involved in this problem are:

(a) To visualise what happens at molecular level.

(b) To use proportionality and mole concept to find number of molecules.

(c) To use statistical average to estimate number of molecules striking unit area of the liquid surface per second.

(d) To use energy conservation principles to find number of molecules evaporating.

4. According to Stefan's Law of radiation, a black body with surface temperature T radiates energy $óT^4$ from its unit surface area every second, where $ó = 5.67´10^{-8}\ W/m^2K^4$ is known as Stefan's constant. If the earth is in radiative equilibrium with the sun and the average temperature of the earth's surface is 300K, estimate the surface temperature of the sun. Take radius of the sun is 7´10^8 m and the mean distance between the earth and that of the sunis 1.5´10^{11} m.

Tasks involved in this problem are:

(a) To estimate energy radiated by sun and fraction of it that is received by the earth.

(b) Energy that is radiated by the earth.

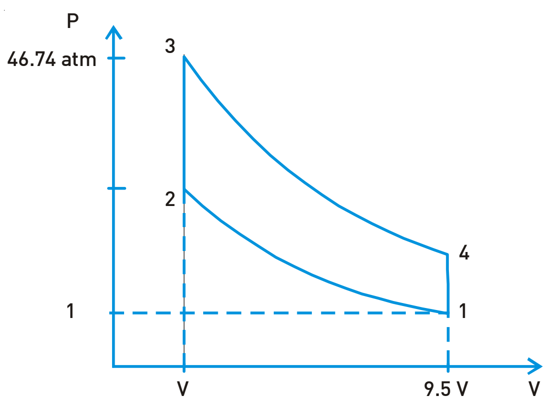
(c) If earth is in radiative equilibrium quantity in (1) and (2) should be equal wich would allow estimation of Sun's surface temperature.

(d) The compression ratio (V_1/V_2) of a four stroke internal combustion engine is equal to 9.5. The engine takes in air and gas fuel at temperature 27°C and pressure of 1 atm. The volume is then compressed adiabatically from state 1 to state 2.

5. The fuel mixture is ignited causing an explosion, which doubles the pressure (state 2 to 3), thus moving the piston into a position in state 3. From state 3 to 4 the gaseous mixture again expands adiabatically until the volume becomes 9.5Vas shown in PV-diagram and the exhausting valve in the cylinder opens up allowing the pressure in the cylinder to return to 1 atm. ($ã = \frac{C_p}{C_V} = 1.4$) Determine

(i) The pressure and temperature of the gaseous mixture in states 1, 2, 3, and 4 respectively.

(ii) The thermal efficiency of the cycle. [IPhO 1976]



An illustrative problem that deals with thermodynamic processes (isochoric and adiabatic).

Tasks involved in this problem are:

(a) To start with initial conditions identifying the processes involved and equations governing them.

(b) To find needed quantities from given information.

(c) To calculate pressure and temperature at various stages of the cycle using gas equations.

(d) To calculate energies involved in different processes to calculate efficiency of a cycle.

Surface Tension

6. Surface tension is exhibited by liquids due to force of attraction between molecules of the liquid. The surface tension decreases with increase in temperature and vanishes at boiling point. Given that the latent heat of vaporization for water L_v = 540 kcal/kg, the mechanical equivalent of heat J= 4.2 J/Cal , density of water $\tilde{n}_{w=10}{}^{3}$ kg l^{-1}, Avogadro's No N_A = 6.0 x 10^{23} k mol^{-1} and the molecular weight of water M_A = 18 kg for 1 k mole.

(a) Estimate the energy required for one molecule of water to evaporate.

(b) Show that the intermolecular distance for water is $d = \left(\frac{M_A}{N_A} \times \frac{1}{\rho_w}\right)^{1/3}$ and find its value.

(c) 1 g of water in the vapour state at 1 atm occupies 1601 cm^3. Estimate the intermolecular distance at boiling point, in the vapour state.

(d) During vaporisation a molecule overcomes a force F, assumed constant, to go fron an intermolecular distance d to d'. Estimate the value of F.

(e) Calculate F/d, which is a measure of the surface tension. [NCERT EP XI]

The tasks in this problem are already listed as (a), (b),... self explainatory. The aim is to construct conceptual understanding through simple calculation.

I : Optics

Learning Objectives

1. To understand formation of image, its nature and position due to mirrors, due to surfaces (plane and curved) separating two media.
2. To understand formation of image, its nature and position due to multiple surfaces and lenses.
3. To understand wave propagation and superposition based on wave nature and path (or phase) difference.
4. To understand mathematical structure dealing with above mentioned points.

Problems

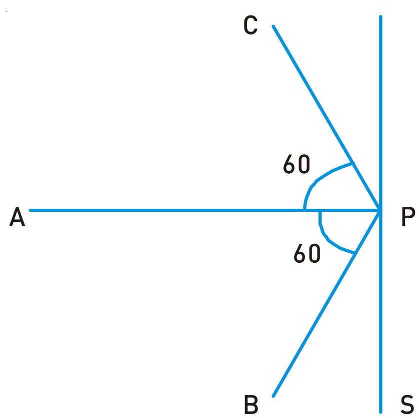
7. Find the height of the shortest plane mirror (held vertically) in which a man six feet tall could see his entire image. At what height above the ground should this mirror be placed in order that the man could see his entire image? *

Tasks involved in this problem are:

(a) Drawing ray diagram to see how image is formed in the mirror.

(b) Geometrically establishing relationship between size of the object and minimum size of the mirror needed.

8. Screen S is illuminated by two point sources A and B. Another source C sends parallel beam of light towards P on the screen (see Fig.) Line AP is normal to the screen and the lines AP, BP, and CP are in one plane. The distances AP, BP, and CP are 3 m, 1.5 m and 1.5 m respectively. The radiant powers of the sources A and B are 90 W and 180 W respectively. The beam C is of intensity 20 W/m^2. Calculate the intensity at P on the screen. . [JEE 82]



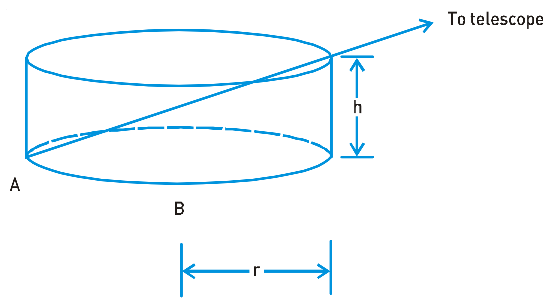
Tasks involved in this problem are:

(a) To recognise difference between point source and a parallel beam of light.

(b) To learn the variation in intensity as light travels distance from different type of sources.

(c) To learn how intensity is related to angle of incidence. This had to be discussed with them.

9. A person looking through a telescope T just sees the point A on the rim at the bottom of a cylindrical vessel when the vessel is empty (see Fig.). When the vessel is completely filled with a liquid (ì = 1.5), he observes a mark at the centre B, of the bottom without moving the telescope or the vessel. What is the height of the vessel if the diametre of its cross-section is 10 cm?



Tasks involved in this problem are:

(a) Identifying incident and refracted rays.

(b) Working out angles for each of the rays.

(c) Application of Snell's law of refraction.

10. An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror. Find the

position of the final image formed. (You may take the distance of the nearer surface of the slab from the mirror to be 1.0 cm) [JEE 80]

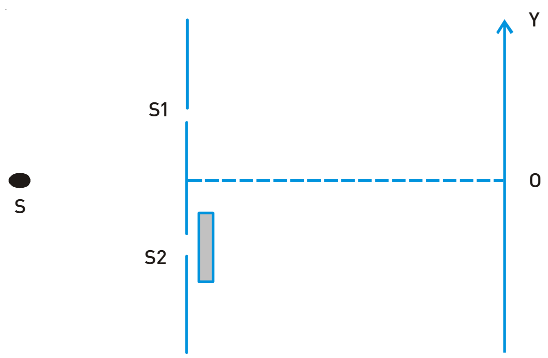
Tasks involved in this problem are:

(a) To draw rays as per the rules of geometrical optics first due to glass slab.

(b) To treat image formed by one as the object for the next and continue.

(c) To locate image by mathematical calculation.

11. The Young's double slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 mm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slit as shown in Fig. P(5) Find the location of the central maximum (bright fringe with zero path difference) on the y-axis. (b) Find the light intensity at point O relative to the maximum fringe intensity. (c) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of light that form maxima exactly at point O. [All wavelengths in this problem are for the given medium of refractive index 4/3. Ignore dispersion] [JEE 99]



Tasks involved in this problem are:

(a) To find the additional path difference introduced by the glass plate.

(b) To locate the point on the screen where path difference will be zero knowing the fringe width.

(c) To estimate the variation in intensity on the screen from consideration of path difference.

Solutions

1. Gas Law

Gas Law: $PV = nRT$: For the gas inside the vessel V and P are constants.

$\therefore nT =$ contstant or $nM_A T = constant \Rightarrow mT =$ constant where m is mass of the gas and T its absolute temperature.

$m_1 =$ 6gm at $T_1 = 7°C = 280$ K
and $m_2 =$ to be found at $T_2 = 147°C = 420$ K

$\therefore m_2 = m_1 (T_1/T_2) = 4\ gm$

$\therefore Dm = m_1 - m_2 = 2$ gm : mass of the gas escaped.

2. Gas Law

$pV = nRT$ and $\frac{pV}{T} = nR =$ constant for a given bubble

$P_{depth} = P_{surface} + h\rho g$ where ρ is the density of water.

$\therefore T_1 = T_2 \frac{P_1V_1}{P_2V_2} = T_2 (\frac{P_1}{P_2})(\frac{d_1}{d_2})^3$ and $P_1 = P_2 + h\rho g$

$T_1 = 283.4\ K = 10.4\ °C$

3. Kinetic Theory

(a) Number of molecules $N = n\,N_A$

where n is number of moles and N_A is

Avogadro Number.

$\therefore \frac{N}{V} = (\frac{P}{RT}) N_A = 2\times10^{19}$ per cm^3

(b) $N/3$ strike wall area l^2 in time $2l/c$ where

$$c = \sqrt{\frac{3RT}{M_A}}$$

$$\therefore \frac{1}{6}\frac{N}{l^3}c = 2.4\times10^{23} \text{ per cm}^2 \text{ per sec}$$

(c) Energy delivered

per cm^2 per sec = $\frac{1}{6}\frac{N}{l^3}c\times\frac{3}{2}kT = 1860.8\ J$

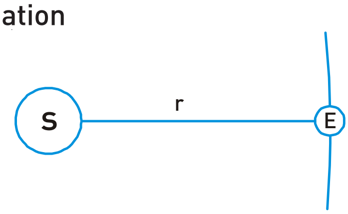
$U/N = 6.7210^{-20}$ J/molecule needed for evaporation.

Thus $\dfrac{\frac{1}{6}\frac{N}{l^3}c\times\frac{3}{2}kT}{U/N} = 5.279\times10^{16}$ gets evaporated every sec from 1 cm^2.

(d) Mean K.E. of vapour molecules = $(3/2)kT$= 7.72´10^{-21} J

Each vaporizing molecule needs energy = 6.72´10^{-20} J

4. Radiation

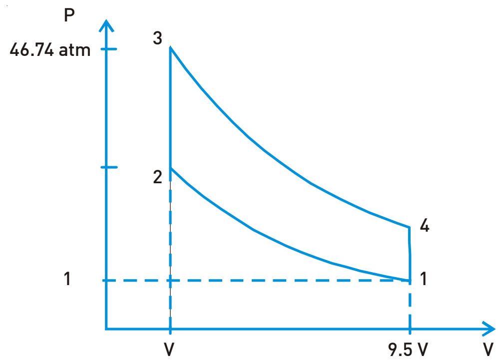


Sun radiates energy sT_S^4 every second. Earth obstructs effectively what is flowing through area pR_E^2 out of $4pr^2$. If Earth's temperature is T_E it radiates 4p$R_E^2 s\, T_E^4$ every second.

∴ In radiativeequilibrium :

$$\sigma T_s^4 \frac{\pi R_E^2}{4\pi r^2} = 4\pi R_E^2 \sigma T_E^4 \Rightarrow T_s = T_E\left(\frac{4r^2}{R_E^2}\right)^{1/4} = 6210.6\text{ K}$$

5. Thermodynamic Cycle



Given: $P_1 = 1$ atm, $T_1 = 300$ K, and $V_1/V_2 = 9.5$

(i) Path 1⟶2 : adiabatic :

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = 23.38atm \quad T_2 = T_1\frac{PV_2}{PV_1} = 738.3K$$

Path 2⟶3 : isochoric $(V_3=V_2)$: $P_3=2P_2= 46.74$ atm

$T_3 = 2T_2 = 1476.5$ K

Path 3⟶4 : adiabatic : $V_4/V_3 = 9.5$ and

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^\gamma = 46.74\times\left(\frac{1}{9.5}\right) = 2.0atm$$

$$T_4 = T_3\frac{PV_4}{PV_3} = 600K$$

(ii) Heat intake = $C_v(T_3-T_2)$=(3/2)R(738.24)=1107.4 R

Heat Exhaust = $C_v(T_4 - T_1)$ = (3/2)R(300) = 450 R

Work done = (1107.4 – 450) R = 657.4 R

∴ Efficiency η= (W/Heat intake)=0.594®59.4%

6. Surface Tension

(a) If L_v is the energy required for unit mass of the substance to evaporate, M_A and N_A are molecular weight and Avogadro Number respectively then energy needed for one molecule to evaporate is

$$u = \frac{M_A}{L_v N_A} = 6.8 \times 10^{-20} J$$

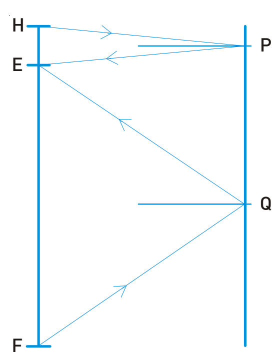
(b) If density of liquid is $\tilde{n}_w$ then volume occupied by a molecule is where $d^3 = \frac{M_A}{\rho_w N_A}$ d is inter molecular distance. This gives $d = 3.1 \times 10^{-10} m$

(c) invapour state this distance increases by a factor of $(1601)^{1/3} = 11.89$ i.e. newintrer molecular distance becomes $d' = 36.3 \times 10^{-10}\, m$.

(d) Thus if intermolecular force per unit distance is F then work done in overcoming it for evaporation is $F(d' - d) = u$. This gives $F = 0.2048 \times 10^{-10}$ N

(e) Thus $\frac{F}{d} = 6.6 \times 10^{-2}\, N/m^2$ is the measure of surface tension of the water.

7. Reflection on a Plane Surface



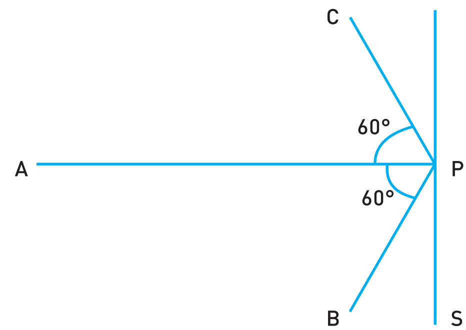
H : Head
E : Eye
F : Face

Refering to Geometry and laws of reflection:
PQ = (1/2) (HE + EF)

= (1/2) (HF)

=(1/2) of height of the person

8. Photometry



A is a point source, and light is incident normally on screen at P :

which gives $I_{P,A} = E_A/d_{PA}^2 = 10/4\pi$ W/m^2

B is a point source, and light is incident at an angle on screen at P: which gives

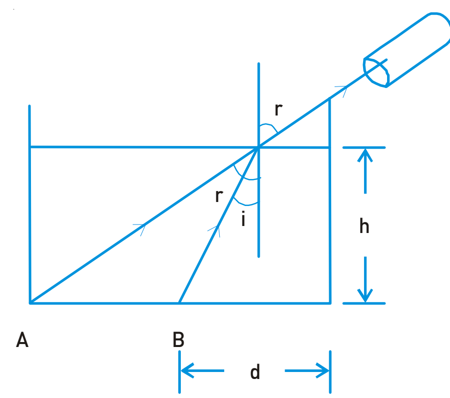
$I_{P,B} = (E_B/4\pi d_{PB}^2) \cos 60° = 40/4\pi$ W/m^2

C sends a parallel beam at P but at an angle

which gives $I_{P,C} = I_C \cos 60° = 10$ W/m^2

$I_P = I_{P,A} + I_{P,B} + I_{P,c} = 12.5/\pi + 10 \approx 14$ W/m^2

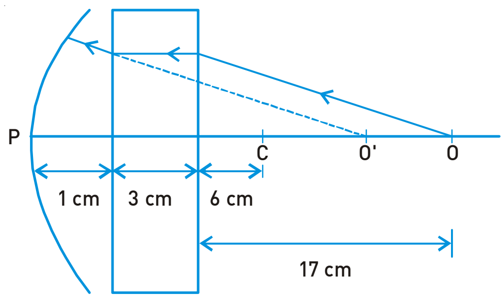
9. Refraction at a Plane Surface



$\frac{\sin i}{\sin r} = \frac{1}{\mu}$, $\sin i = \frac{d}{\sqrt{h^2 + d^2}}$ and $\sin r = \frac{2d}{\sqrt{h^2 + 4d^2}}$

Solving which we get $h = d\sqrt{\frac{4\mu^2 - 4}{4 - \mu^2}} = 8.45\,cm$

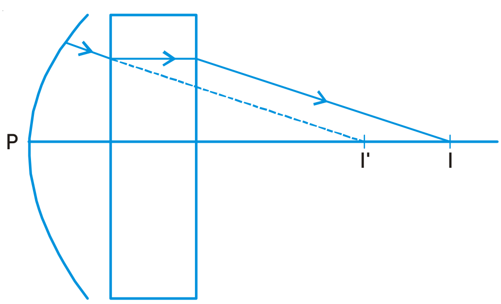
10. Refraction and Reflection at a Curved Surface



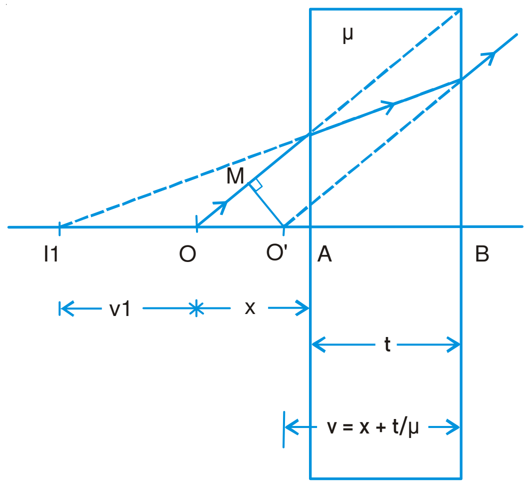
As seen from the mirror, if object is at O, it appears to be at O'. The displacement can be given by OO' = $t\,(1 - 1/\mu)$ = 1 cm [* This is proved as given at the end]

$\Rightarrow$ PO' = PO – OO'' = 20 cm

For mirror $f = R/2$ = 5 cm and u = 20 cm and using $\Rightarrow v$ = PI' = 20/3 cm



and II' = $t\,(1 - 1/\mu) = 1$ cm as above which gives PI = 23/3 cm = 7.67 cm from the mirror.



For a curved surface separating two media

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

here R = ∞ for both the surfaces

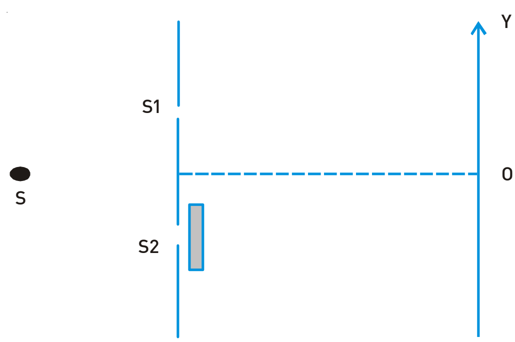
$\therefore$ For 1st surface $v_1 = -\mu x$

$\therefore$ For 2nd surface $u = -(\mu x + t)$

$\therefore \frac{1}{v} + \frac{\mu}{\mu x + t} = 0 \Rightarrow v = -(x + t/\mu)$

$\therefore$ OO' = OB – O'B = $(x + t) - (x + t/\mu) = t(1 - 1/\mu)$ = 1cm

11. Young's Double Slit Interference



All wavelengths here corresponds to $m = 4/3$

(a) Fringe width $\Delta x = \lambda D/d = 2$ mm

$t(\mu - 1) = n\lambda$ and $\mu = {}_m\mu_g = 9/8$

$\therefore n = t(\mu - 1/\lambda) =$: no of fringes

$\therefore$ Shift $y = n\Delta x = ´2 =$

(b) $\Delta x = \lambda/6 + 2\lambda \Rightarrow$ Phase shift $a = 2p\Delta x/l = \pi/3$: acute angle

$\therefore I_o = 4A^2$ and $I = A^2(1 + \cos\alpha)^2 = (9/4)A^2 = (9/16)I_o$

(c) $n = t(\mu - 1/\lambda)$: integer : $n = \dfrac{1.3 \times 10^{-6}}{\lambda} = \dfrac{1300}{\lambda}$

for λ in nm.

$\therefore n = 2 \Rightarrow \lambda = 650$ nm

and $n = 3 \Rightarrow \lambda = 433.3$ nm in given range.

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