# EXPLORING MAHTEMATICS THROUGH ORIGAMI

**O.P. Gupta**
PGT, Maths
B.I.O.P. Sr. Sec. School
Kirandul, Dantewada
Chhattisgarh

Origami literally translates as Ori (folding) gami (paper). It could be used as a valuable method for developing vital skills in mathematics. It is not just a game. All general principles of mathematics are involved in creation of shapes through origami. It can be easily expressed with the help of folding. By using simple square sheets of paper, students can use their imagination and creativity. Difficult and abstract mathematical concepts can be explained in an interesting manner through origami.

The basic tool of origami is a simple sheet of square paper. We will fold a square sheet of paper in any way and begin to see the unlimited possibilities that the square has to offer.

Why is the square sheet of paper used in exploring mathematics in folding?

The square sheet of paper used due to its some peculiar properties:

1. It has all properties of quadrilateral, parallelogram, trapezium and rectangle.
2. Its diagonals are equal in length and bisect each.
3. Its diagonal divides the shape into two isosceles right triangles.
4. The interesting point of diagonals lies on angle bisector of all angles and perpendicular bisector of all sides.
5. All its exterior angles and interior angles are equal.

## Objectives

By folding a square sheet of paper, different shapes can be formed. It may represent various aesthetic views of nature. It is done under the scope of ORIGAMI that discloses the hidden ideas of mathematics, when the folded paper is unfolded. Folding and unfolding of the square sheet of paper can be an unending process, which may lead to a new dimension in the teaching of mathematics. Since, this process connects the subject with beauty and real life problems in a practical way, the subject becomes not only creative but also interesting to the teacher and as well as the taught.

**I. Folding a Square Sheet of Paper into Half:** There are so many ways to half-fold a paper. Every folding creates a number of geometrical shapes. Some of them are described below:

1. **Diaper fold:** A fold made along the diagonal of a square sheet of a paper is called a diaper fold. A diaper fold reveals the shape of two triangles, and the same is shown in Fig.1. Diaper fold is mathematically known as angle bisector fold of a square sheet of paper.

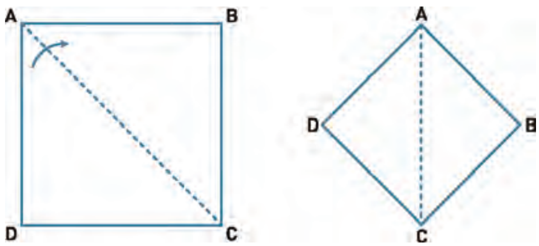


Fig.1

## Mathematical Facts Revealed in Diaper Fold

- The triangles formed are congruent.
- The triangles are isosceles right triangles.
- The triangles are equal in area.
- Area of each triangle is half of the area of the square sheet of paper.

2. **Cupboard fold:** A fold made through the perpendicular bisector of any of its side is called a Cupboard Fold. And the folded square sheet of paper turns into a rectangle.

A second cupboard fold, through yet another perpendicular-bisector of the same side, demonstrates the shape of yet another rectangle, but half the size of the first folded rectangles.

And on unfolding, when the paper is seen through its folded-creases, it appears to be divided into four equal parts.

Cupboard fold is mathematically known as perpendicular bisector fold (of any side) of a square sheet of paper. The whole process is shown in Fig. 2.

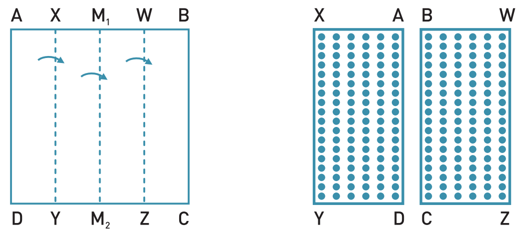


Fig.2

## Mathematical Facts Revealed in a Cupboard fold

- The two rectangles formed by the first cupboard-fold are equal in area, with each rectangle as exactly half the square sheet of paper. The ratio of the sides of the rectangles so formed is 1:2.
- The four rectangles formed by the second cupboard-fold, are also equal in area, with each rectangle as exactly one-fourth of the square sheet of paper. Add the ratio of the sides of the rectangles so formed is 1:4.

3. **Blintz fold:** To make this fold, the square sheet of paper is first half-folded to mark the mid-point of all its sides. Once the mid-points are marked, the mid-points of the adjacent sides of the square sheet of paper are joined by further folding, such that the square sheet of paper is turned into a smaller size square. Blintz fold is therefore obtained by joining the mid-points of adjacent sides, by folding the given square sheet of paper. The whole process is shown in Fig.3.

## Mathematical Facts Revealed in a Blintz fold

- The area of the small square obtained through this fold is half of the area of the original square sheet of paper.
- The ratio of the sides of the small square so obtained to that of the original paper is $\sqrt{2}:2$ (wherein the value of $\sqrt{2}$ is approximately 1:4).

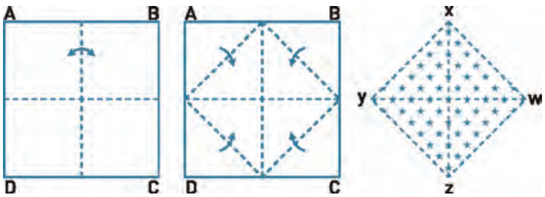


Fig. 3

## House - roof fold (Combined with Cupboard-fold)

The square sheet of paper is first half-folded. And then, two adjacent corners are folded to fall on the central crease formed by the half-fold, such that two adjacent isosceles triangles formed on one-half of the square sheet of paper. Now, this half with two adjacent triangles, looks like a house-roof. Next, the other half is a half-folded such that one side of the square falls on a side of both the triangles. The entire process is demonstrated here by Fig.4. The folded paper finally appear with five sides, i.e., as an irregular pentagon (Fig.4.).

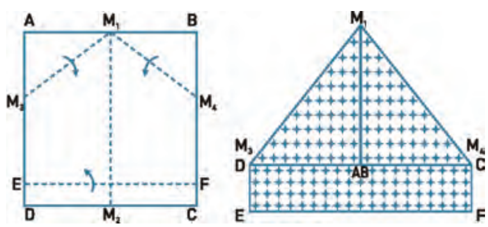


Fig.4

## Mathematical Facts Revealed in a House-fold (combined with Cupboard fold)

- The pentagon so formed will have two adjacent sides and two opposite sides as equal.
- Three internal angles of the pentagon will be 90° each and the other two will be 135° each.
- The area of polygon thus obtained will be half the area of the square sheet of paper.
- The folded polygon consists of two isosceles right triangles and one rectangle.
- The equal sides of right triangles are half of the side of square sheet of paper.
- The area of each right triangle will be one-eighth of the area of square sheet of paper.
- The length and breadth of rectangle has ratio of 4:1.
- The area of rectangle is one-fourth of the area of the square sheet of paper.
- The area of rectangle will also be equal to sum of the areas of the two isosceles right triangles.

**II. Folding a Square Sheet of Paper into Less or More than Half of its Area:** There are also several ways to fold the square sheet of paper to convert it into either less or more than half of its area. Different geometrical shapes thus obtained are explained as follows:

**1. House-roof fold:** The square sheet of paper is first half-folded. Then, two adjacent corners are folded to fall on the central crease formed by the half-fold, so that the paper now looks like a house-roof. Now, the folded paper has five sides, i.e., like an irregular-pentagon. The entire process is demonstrated through Fig. 5.

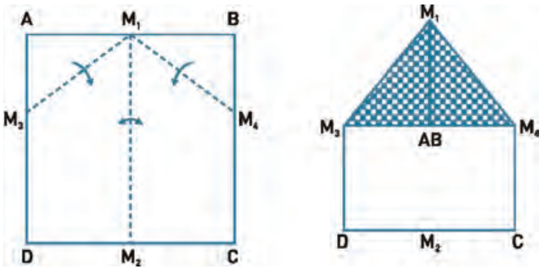


Fig.5

## Mathematical Facts Revealed in a House-roof fold

- The pentagon so formed will have two adjacent sides and two opposite sides as equal.
- Three internal angles of the pentagon will be 90° each and the other two will be 135° each.
- The area of polygon thus obtained will be three-fourth of the area of the square sheet of paper.
- The folded polygon consists of two isosceles right triangles and one rectangle.
- The equal sides of right triangles are half of the side of square paper.
- The area of each right triangle will be one-eighth of the area of square sheet of paper.
- The length and breadth of rectangle has a ratio of 2:1.
- The area of rectangle is half the area of the square sheet of paper.

**2. Reverse fold:** The square sheet of paper is first folded into a diaper fold and then two opposite corners of the diagonal-crease are folded in a manner such that both the opposite sides coincide with diagonal, and the final shape looks like a kite (Fig.6).

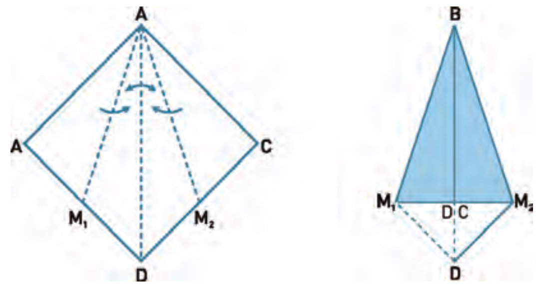


Fig.6

## Mathematical Facts Revealed in Reverse fold

- Reverse fold divides the square sheet of paper into four triangles. The two triangles formed in inner region are congruent. Similarly, parts of triangle in outer region of square sheet of paper are also congruent.
- The area of triangle in inner region is $\sqrt{2}$ times the area of triangle in outer region.
- The second and third fold of square sheet of paper divides the side of the paper into $1:\sqrt{2}$ ratio.
- The area of kite is more than half of the area of square paper, i.e., the area of kite is $3/5^{th}$ of the area of square sheet of paper.
- The four interior angles of kite (quadrilateral) are 45°, 112½°.

**3. Squash fold:** The square sheet of paper is first folded into half and unfolded. This fold made through the perpendicular bisector of any of its side. Again, square sheet of paper is folded in half by another side. Now top right corners are folded forward and backward in such a way that the corner coincides the fold. The final looks like a trapezium (Fig.7).

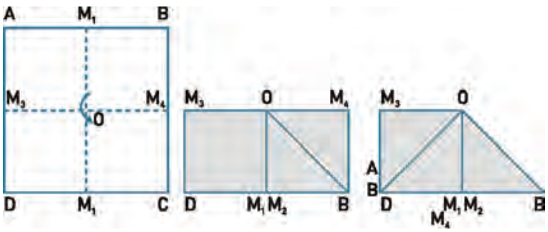

Fig. 7

## Mathematical Facts Revealed in Squash fold

- The four interior angles in the trapezium are 45º, 90º, 90º, 135º and parallel sides are in 1:2 ratio.
- The trapezium has right triangle and square shaped figures (or set of three right triangles, among two of them has common hypotenuse).
- In 1st case, we observe that the right triangles are isosceles and side of square is also equal to sides of isosceles right triangle. In second case, we find all three right triangles are isosceles and congruent triangles.
- Thus, the area of trapezium is either equal to the sum of areas of isosceles right triangle and the area of square of half side of square sheet of paper (considering 1st case) or the sum of area of three isosceles right triangles (considering 2nd case). It can be verified by following the area of trapezium.
- The area of above Trapezium - $\frac{1}{2} \times (X+2X) \times X$, where X is the half of the side of the square sheet of paper. So we can conclude that the area of trapezium = $3/8 \times a$, where a is the side of the square.

**4. Squash fold (Combined with House-roof fold):** If the square sheet of paper is folded by squash fold, again the free end of the paper is folded by House-roof fold. The folded paper finally appears in a parallelogram shape (Fig.8).

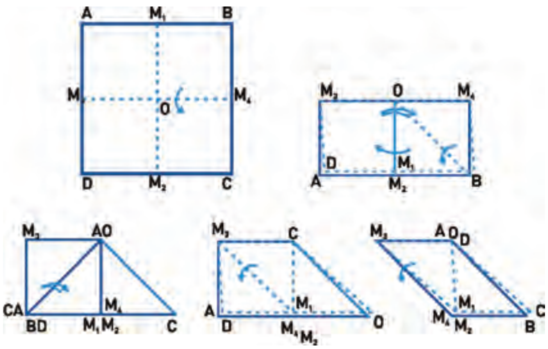

Fig. 8

## Mathematical Facts Revealed in a Squash (combined with House-roof fold)

- The parallelogram so formed will have two adjacent sides in $1:\sqrt{2}$ ratio in lengths.
- The pairs of opposite angles are of 45º and 135º.
- The area of parallelogram is 1/4th the area of the square sheet of paper.
- The ratio of lengths of shorter and longer diagonals of parallelogram is $1:\sqrt{5}$.

**5. Pinch fold:** To make this fold, the square sheet of paper is first folded along its each diagonal (i.e., two diaper fold). After unfolding, we get four isosceles triangles in it. Again we fold the square sheet of paper into half along its sides. The folded creases will be the altitudes of the isosceles triangles. Now we fold all adjacent perpendicular bisectors of altitudes and lap the corner on each side of square and crease. This fold is known as Pinch fold. The steps of the fold are explained by the diagram in fig.9.

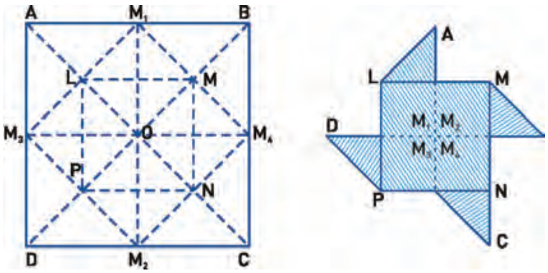


Fig. 9

## Mathematical Facts Revealed in a Pinch fold

- The shape obtained by above folding is square with four isosceles right triangles at its four corners.
- The area of square found after folding is 1/4 and area of each right triangle is 1/32 of original square sheet of paper.
- The area of whole shape is 9/32 of the area of original square sheet of paper.
- The sum of areas of four isosceles right triangles is half of the area of its square.
- The area of shape obtained after folding is less than the area of the original square sheet of paper.

**6. Petal fold:** The square sheet of paper is first folded into half by diaper fold and unfolded to get a centre line. Now right and left corners are folded to the centre line. This is primarily known as reverse fold. Again both opposite corners of the diagonal-crease are folded so that both remaining opposite sides coincide with the diagonal and final shape looks like a rhombus (Fig.10 and Fig.11).

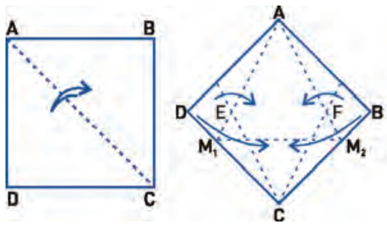


Fig. 10

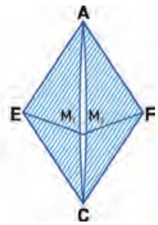


Fig. 11

## Mathematical Facts Revealed in Petal fold

- The length of longer diagonal of rhombus found after fold is equal to the length diagonal of the square sheet of paper.
- The length of shorter diagonal of rhombus is less than half of its longer diagonal. The ratio of length of its diagonals is 1:2 (approximately).
- The area of the rhombus is slightly less than half of that of the square sheet of paper i.e., the ratio of areas of rhombus and square sheet of paper is 1:2 (approximately).
- The angles of rhombus are $40^o$, $135^o$, $45^o$ and $135^o$.