Developing Geometrical Reasoning among Children at Secondary Level

K. Sreedharacharyulu[*] and Sardar Dharmendra Singh[**]
Email: [*]sreedhar.456321@gmail.com and [**]dschahel@gmail.com

Abstract- *Mathematics is the most beautiful subject among all of the subjects in the school curriculum. The beauty of mathematics can be enjoyed because of its nature of having reasoning. The beauty is enriched even more when we deal with geometry. Geometrical reasoning is not only useful for making difficult problems solving easy but also it gives pleasure to human brain to have more joy. The present paper is theoretical paper which discusses on classroom strategies in developing geometrical reasoning among children at secondary level. These strategies may be project based teaching learning process, ICT based teaching learning process (especially with geogebra) and digital games like "Police squad" and some hands on activities etc.*

Key words: *Reasoning, geometry, geometrical reasoning, secondary level, teaching learning process.*

Introduction

There is geometry in the humming of the strings; there is music in the spacing of the Spheres- Pythagoras as famously quoted by the Greek philosopher mathematician, Pythagoras, mathematics is very beautiful and pleasant and geometry adds more beauty to it. Geometry can be viewed and felt everywhere in our daily life viz. in our house, farm, roads, temples, objects etc. We may handle geometry intuitionally or by unknowing reasons so many times. For example a farmer uses many geometrical ideas while farming. Children use so many geometrical ideas intuitionally while playing. But, the question arises here is…why do students feel difficulty in learning geometry when it comes to learning geometry in classroom situations?

When we look into the meaning of geometry and reasoning in it and try understanding it, we may find the reason and solutions for the above problem.

Geometry: Meaning

According to Oxford dictionary, the meaning of geometry is stated like this It is "the area of mathematics relating to the study of space and the relationships between points, lines, curves, and surfaces". And **As per Merriam- webster dictionary**,

It is "a branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, surfaces, and solids". On the whole, when we try to understand the meaning of geometry, we may conclude like this.

1. It is a branch of mathematics
2. It is study of spatial understanding
3. It explains about properties and extracts relationships between points, lines, angles, curves, surfaces etc.

Meaning of Reasoning

It is "the process of thinking about something in order to make a decision"- Cambridge dictionary When we try to understand the meaning of reasoning, It is one of the best forms of controlled thinking consciously towards the solution of a problem. It is realistic in the sense that the solution is sought always in reference to the reality of the situation. We can solve many problems in our day-dreams, dreams and imaginations but they are unrealistic solutions. As Sherman defined, "reasoning is a process of thinking during which the individual is aware of a problem identifies, evaluates, and decides upon a solution". Reasoning is used not only when we want to solve an immediate problem but also when we anticipate future problems. Reasoning plays a significant role in one's adjustment to the environment. It not only determines one's cognitive activities but also influences the behaviour and personality.

Definitions of Reasoning

- "Reasoning is a stepwise thinking with a purpose or goal in mind" —Garrett.
- "Reasoning is the term applied to highly purposeful, controlled and selective thinking"—Gates.
- "Reasoning is the word used to describe the mental recognition of cause and effect relationships, it may be the prediction of an event from an observed cause or the inference of a cause from an observed event"—Skinner.

Thus reasoning is a highly specialized thinking which helps an individual to explore mentally the cause and effect relationship of an event or solution of a problem by adopting some well-organized systematic steps based on previous experience combined with present observation.

Types of Reasoning:

Reasoning may be classified into two types.

1. Inductive reasoning

It is a specialized thinking aimed at the discovery or construction of a generalized principle by making use of particular cases, special examples and identifying of elements or relations. For example, Mohan is mortal, Radha is mortal, Karim is mortal; therefore, all human beings are mortal.

2. Deductive reasoning

It is the ability to draw some logical conclusions from known statement or evidences. Here one starts with already known or established generalized statement or principle and applies it to specific cases. For example, all human beings are mortal you are a human being, therefore, you are mortal.

3. Conditioned reasoning

It is the reasoning tied down by some specific condition such as the following. For example, if there is a solar eclipse, the street will be dark. There is a solar eclipse…The streets are dark.

4. Categorical reasoning

This type of reasoning is based on some categorical statements. For example, all Robins are birds. All birds lay eggs…. All Robins lay eggs.

5. Linear reasoning

This type of reasoning involves straight forward relationships among elements. For example, If Ram is taller than Mohan and Mohan is taller than Sohan, Ram is the tallest.

6. Geometric reasoning

Geometric reasoning is the use of critical thinking, logical argument and spatial reasoning to solve problems and find new relationships. Students must first have a critical understanding of any underlying assumptions and relationships. This allows them to develop coherent knowledge and apply their reasoning skills.

The purpose of geometric reasoning is to determine results from previously established truths and to then apply these results in the solution of problems. It can also be used to verify or prove results.

Visualisation plays a critical role in geometric reasoning. An 'image in the mind' provides students with the necessary structures of a shape in order to define and classify them and then deduce other properties from them. Convincing others through a common mathematical discourse is seen as a necessary component in the meaning-making process of geometry. It is necessary for teachers to have the pedagogical content knowledge necessary to develop and support geometric argument in the classroom. For many children, the learning of geometry starts before schooling with the recognition of shapes based on their appearance (Kilpatrick, Swafford and Findell, 2001). This then develops throughout primary schooling with the identification of shape properties. Associated measurement concepts also begin quite early when a number is assigned to continuous quantities (Browning, Edson, Kimani and Aslan-Tutak, 2014). Many relationships between lengths or areas that students encounter in their mathematics classroom do not depend upon measuring, but on their knowledge of geometric structures. For example, because rectangles have four right angles, they have two pairs of parallel sides; since parallelograms have two pairs of parallel sides, they have equal opposite angles. These results

are established without the aid of rulers and protractors. Establishing geometric relationships requires thinking and reasoning about properties of shapes and their diagrams (Cooke, 2007). Geometry is an excellent vehicle to develop measurement and number concepts, and broader reasoning skills at all levels of learning.

Rationale

Geometry is an undervalued part of mathematics, often seen simply as shape recognition, exploring visual patterns and very little else (Jones, 2002). Given the potential for geometry to develop students' spatial abilities, reasoning skills, and abilities to solve real-world problems (Jones and Mooney, 2003; Marchis, 2012), there is a strong need for developing specific teaching and learning opportunities. These opportunities should engage all students in activities that enhance visualisation and geometric reasoning. Spatial ability provides us with capacities to perceive the visual world (Gardner and Hatch, 1989). However, there is a basic reluctance of students to use visualisation in ways to communicate geometric concepts.

Objectives of the Study

- To study innate abilities of students in developing geometrical reasoning
- To study individual differences among children in developing geometrical reasoning.
- To propose a pedagogical process to develop geometrical reasoning.

Methodology

Innate abilities of children:

Competency is the ability to act, having the capacity. Everybody can "talk" but the ability is to get the required information from others by asking them. Although, everyone knows writing but writing an application for one's own need is the ability. So, in this context one should understand the competencies or abilities. The knowledge which is acquired up to the level of skillful performance is considered as ability. Generally, abilities will be according to the nature of the subject. These are psychological. In school level, the children learn different. Indeed deciding what the children has to achieve? What to perform? by learning these subjects, is very important. Therefore, the abilities, to be achieved, are determined by the nature of various subjects in different classes. So, it our responsibility to look after the children to attain them. Accordingly, new textbooks have to be bought and utilized in a proper manner. That means, we need to look after that the children has to solve the exercises, which are based on the abilities, given at the end of the lessons, by themselves. Prohibit the habit of copying from the guides and question banks, which hinders the development of abilities among the children. They will develop the competencies by thinking, tryouts, and by studying. And also, by participating, and writing on their own etc., In this regard, the teachers are required to conduct the teaching - learning activities by preparing the year plan, and lesson plans well in advance. Every child has his own inherent talents, and abilities. So, the chief aim of education is to bring out these innate powers and inherent abilities and to develop them. The innate abilities of the children: These innate

abilities are exist in every child. They will be developed in different children in different ways. So, the important duty of the school is the complete development of these powers, capabilities and competencies and prepares them as good citizens.

Such innate abilities are:

- Thinking
- Observing comparing and collecting.
- Prediction
- Identifying the differences and saying
- Classifying and computing
- Estimating, or assessing
- Cause and effect, formulation
- Questioning, forming hypothesis
- Giving reasons
- Establishing rules and generalising
- Analysing
- Synthesizing
- Creating / producing
- Playing, singing, acting

The main functions of reasoning are considered to be to understand, to explain and to convince. A teacher should give the freedom to students in the class to generate conjectures experimentally. When a conflict arise and the class split into two or more groups, debating different conjectures within the same situation, the pupils had the freedom to choose various kinds of arguments in order to try to convince each other. The arguments were judged by their power of conviction. This pedagogical approach reflects a considerable ongoing change in the mathematics education community, towards reasoning in mathematics as a whole, towards reasoning in geometry, and, in particular, towards proving and until the students achieves the competence of proving a theorem or a rider.

Developing geometrical reasoning

Geometry involves three kinds of **cognitive processes** which fulfil specific epistemological functions:

Visualisation Processes with regard to space representation for the illustration of a statement, for the heuristic exploration of a complex situation, for a synoptic glance over it.

Construction processes by tools construction of configurations can work like a model in that the actions on the representative and the observed results are related to the mathematical objects which are represented; - **reasoning** in relationship to discursive processes for extension of knowledge, for proof, for explanation. These different processes can be performed separately. So

visualisation does not depend on construction: there is access to figures, whatever way they are constructed. And even if construction leads to visualisation, construction processes depend only on connections between mathematical properties and the technical constraints of the used tools. Ultimately, if visualisation is an intuitive aid that is sometimes necessary for finding a proof, reasoning depends exclusively on the corpus of propositions (definitions, axioms, theorems) which is available. And in some cases visualisation can be misleading or impossible. However these three kinds of cognitive processes are closely connected and their synergy is cognitively necessary for proficiency in geometry.

High school curriculum consists of Euclidean geometry and proofs. Many students find it boring to learn. At secondary level we expect students to do formal deductive reasoning. But often students do not understand basic concepts and ideas as they don't have enough experience with geometric reasoning activities. The five levels of geometric thinking are Visualization, Analysis, Abstraction, Deduction, Rigour

Suggested process of developing Geometrical reasoning

1. Understanding the concept of shape (Visualization)

- A shape is determined by its properties.
- Transformations like rotation, translation, or reflection do not change the properties and hence does not change the shape.
- A deformation that changes the properties of the shape changes the shape as well.

2. Analyzing and describing shapes (Analysis)

- Shapes have many attributes and properties - for example: straight sides, curved sides,
- equal sides, right angles, obtuse angles, acute angles, reflex angles, parallel sides etc.
- Knowing and understanding these properties help us describe, classify and reason about shapes.
- It is important understand shape properties like number of sides, types of angles present etc. and use them as accurately as possible while describing, classifying or reasoning about shapes.

3. Defining and classifying shapes (Abstraction)

- A set of shapes could have one or more common properties, and sometimes no common properties.
- A class of shapes is defined by the common properties shared by each member of the class. For example: All closed figures with exactly three straight sides form a class of shapes - we call them 'triangles'.

4. Developing language and notations of geometry

After knowing definitions and properties, students should develop the competence of making statements and show them in the form of mathematical notations appropriately.

5. Property-based inferential reasoning (Deduction)

This is a critical stage, where students step into the realm of inferential if-then reasoning, with a lot of conjecture making, testing and informal reasoning activities.

6. Understanding the need for proof (Rigour)

In this final theme, students come up with hypotheses through inductive reasoning, verify them, but also learn why verifications do not count as 'proofs', and why deductive proofs are sometimes necessary.

Pedagogical process- an example

Let us observe an example of developing geometrical reasoning

Aim: to prove each angle in an equilateral triangle is 60 degrees

1. Understanding the concept of shape (Visualization)

The students are supplied different types of polygons and make them to understand "triangle"

2. Analyzing and describing shapes (Analysis)

The students will differentiate the properties of triangles from other polygons and analyse the properties of triangles. One of the properties of the triangles is "Sum of all angles in a triangle is 180 degrees"

3. Defining and classifying shapes (Abstraction)

The students will be supplied different types of triangles viz. Equilateral, Isosceles and Scalene, and classify them as per their properties.

4. Developing language and notations of geometry

Language of geometry will developed at this stage and they are made to express sides and angles in mathematical notations as side AB, side BC and side CA in triangle ABC and similarly angles.

5. Property-based inferential reasoning (Deduction):

Discussion

- What is given type of triangle?
  Equilateral
- What are properties of the equilateral triangle?
  All sides are equal and also all angles are equal
  AB=BC=CA that implies angle A= Angle B=Angle C
- What is the sum of interior angles in a triangle?
  180 degrees

AngleA+AngleB+AngleC=180 degrees

- What can you conclude from above two statements?
  AngleA+AngleA+AngleA=180 degrees
  3 (Angle A) =180 degrees
  Angle A =60 degrees

6. Understanding the need for proof (Rigour)

Students are finally asked to this in the formal mathematical language. And they are asked prove this in other ways.

Conclusion

'Learning' is a social process. Children learn by observation, directly or indirectly, while talking to people, while discussing and working with the group. It may happen incidentally. But, in general, elders and parents feel that children learn only by their instructions. When we observe the things like, what to tell to the children? Whether they learn only from you? They know only what we have explained or told? How come they know the other things? . Then we understand that how to develop mathematical reasoning in a child. By discussing with others they expand their knowledge. Such co-operative or collaborative learning helps to develop participatory learning. Hence students develop mathematical reasoning by using their innate abilities.

This theoretical paper suggests a pedagogical process of developing geometrical reasoning among students at secondary level. This paper suggests a teacher the process of geometrical reasoning can be developed. It is also suggested to implement innovative strategies of teaching and learning processes like ICT utilisation, laboratory, question-answer discussion (Socratic method) etc.

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