Difficulties in Graphing Quadratic Functions

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Abstract- *The purpose of the present study was to investigate the difficulties of 10^{th} - grade students in graphing quadratic functions (leading coefficient 1).60 students from 10^{th} grade of my school constituted the sample of the study. In data collection, 2 questions related to graphing quadratic functions were conducted to all students. The students' errors were listed from their answers. The findings revealed that some of the students didn't know the properties of quadratic functions. some of the students did mistakes in the calculation table because of their weakness in integer multiplication and squaring integers.*

The researcher developed remedial activities based on the results. During the remedial classes' students were given enough drill on integer multiplication and integer addition rules which lasts for 20 days. They were also encouraged to find patterns in the first difference of the quadratic sequence to cross check the calculation table and mistakes in the graph. At last some students can plot points in the graph even without the calculation table. At last Post test was conducted and the results were compared with the pre-test.

Key Words: Graphing quadratic functions, parabola, patterns in quadratic sequence, quadratic equation.

Introduction

Algebra is one of the main branches of mathematics education and important in every area of life. With the abstract thinking structure, it provides, in many respects, it undertakes a common bridge and language function conceptually and theoretically among the sub-areas of mathematics and elements of other disciplines (Erbaş, Çetinkaya and Ersoy, 2009). Quadratic expression, Quadratic function and Quadratic equation have very important place in the learning area of algebra. Quadratic equations can be solved in many ways such as:

1) Factoring method.
2) Quadratic formula.
3) Completing the square method and
4) Graphing method.

To obtain the roots of a quadratic equation $ax^2 + bx + c = 0$ graphically, students have to draw the graph of quadratic function $y = ax^2 + bx + c$.

The concept of function seeming to be related with algebraic demonstration is actually very much related with graphs. For Functions are pictured with graphs and can be perceived and interpreted visually (Bloch, 2000; Nadia, 2004; cited in Yavuz and Kepceoğlu, 2010). In this paper we are going to see the various difficulties faced by my students in graphing a quadratic function, mistakes done by them and whether the remedial measures planned and implemented by the researcher helped them to overcome the difficulties?

Objectives

- To make students able to draw a parabola in the form $y = x^2 + bx + c$.
- (leading co-efficient 1).
- To make them interpret the important properties of parabolic graphs.
- To identify the students who are finding difficulties in graphing a quadratic function using a pre-test
- To collect the list of errors done by them in the calculation table.
- To collect the list of errors done by them in drawing parabola.
- To analyze the errors collected from the pre-test data.
- To develop the remedial measures based on the results.
- To implement the remedial measures effectively.
- To test the achievement after the remedial measures using a post test.
- To compare the results obtained from the post- test with the pre-test.

Rationale

- Understanding the properties of a parabola and graphing a quadratic function helps the students very much in their higher secondary studies.
- Scope for developing basic arithmetic skills and graphing skills.
- In 10^{th} grade, we are having the following types of problem. To solve those problems, they should know how to draw the quadratic graph
- Discussing the nature of solutions of a quadratic equation graphically.
- Example :Discuss the nature of solutions of the quadratic equation $x^2 + x - 12 = 0$
- Solving quadratic equations through intersection of lines.

Example: Draw the graph of $y = x^2 - 4x + 3$ and use it to solve the equation $x^2 - 6x + 9$ Late bloomers have to solve the above problems in order to pass in the examination So it is very important to make every student to do those type of problems.

Project site

To conduct the research 60 students of class 10 of Government Higher secondary school, Mangudi which is located in a rural area in Sivagangai district, Tamilnadu. The age group is 15-16.

Statement of Hypothesis

- What are the difficulties do my students have in graphing a quadratic function?
- What are the skills required in order to draw a graph without any mistake?
- what are the remedial measures that can be implemented?
- Whether the remedial measures will help them or not?

Methods and procedures

First, I conducted a pre-test to my class students having strength 60 which consists of the below questions.

- Draw the parabola for the quadratic function $y = x^2 - 5x - 6$
- Draw the parabola for the quadratic function $y = x^2 + 3x + 2$

These two questions are intended to test their ability of graphing quadratic functions.

In this study we are going to see quadratic functions with leading co-efficient +1 only. In the first question, the co-efficient of x is negative and in the second problem the co-efficient of x is positive in order to test their integer multiplication ability. After completion of the test, the results were analyzed and the mistakes done by them were categorized.

The analysis includes the possible causes for those mistakes and the suitable remedial activities that can be performed in order to help them.

The remedial activities were planned based on the results from the pre-test. Worksheets were prepared as seen in annexures. It took 20 days to complete the remedial activities. Finally, the post test was conducted and the results were compared with the results of the pre-test.

Results

The mistakes done by them are categorized into two types:

1. *Mistakes in the calculation table.* 2. *Mistakes in the graph*

Students mistakes pictures can be seen in Annexure 1.

Table: 1 Mistakes in the calculation table-pre test

S.No.	Topics	Mistakes	Frequency	% of students
1	Mistakes when calculating x^2	1. Putting "-"sign when finding X^2. Examples: $(-3)^2 = -9$ $(-2)^2 = -4$	8	17%
		2. Wrong calculation of X^2 Examples:	7	15%

		$(3)^2 = 6 \quad (-4)^2 = 8$		
2.	Mistakes when calculating bx	1. Sign mistakes -5(-2) = -10 3(-2) = 6	7	15%
		2. Multiplying co-efficient of x with x^2 values instead of x	8	17%
		3. Multiplying co-efficient of x with values of x Wrongly. Example: -3(-2) =9 5 x 0 =-5	8	17%
		4. Treating co-efficient of x as constant For -5x writing -5 in the entire row.	5	12%
3.	Mistakes in values of y	Wrong calculation when adding values of $x^2 + bx + c$ Example: 4-10-6=12	16	30%
4.	Mistakes in ordered pairs	1. when writing as an ordered pair interchanging x and y values. 2. Missing the signs when writing as an ordered pair. 3. writing wrong set of values as an ordered pair.	9	18%

Table: 2 Mistakes in the Graph-Pre Test

S.No	Topics	Mistakes	Frequency	% of students
1.	Graph	No curve: 1. Using ruler scale to join points.	3	5%
		2. Drawing curves not smoothly.	15	25%
	Scaling in Axes	Incorrect scaling: 1. Skipping one unit from origin in axes and marking x or y values.	2	3%

2.		2. Taking 1cm = 1 unit in one side of the axis and 1cm=2 units in the other side.	4	7%
3.	Plotting points	Incorrect plotting of points: 1. Confusion in Plotting order Pairs. Example: Plotting (-2,0) as (0, -2)	7	12%
		2. Points were plotted in the wrong quadrants.	12	20%
		3.points were plotted completely which are not in the table.	8	13%

Discussion

The mistakes were analysed to know the possible causes and the methods to reduce the mistakes. As seen in the Table1 students did mistakes mostly calculating 'bx' and values of y.This shows their weakness in integer multiplication rules and addition rules.They are also weak in squaring integers.They are confusing whether to multiply by 2 or by the number itself.If we make them good at those skills, majority of the mistakes will be reduced.Its very important to make the students good at calculation table so that they can draw the graph without any mistake.

As far as table 2, most of the mistakes were due to plotting of points wrongly and drawing the curves not smoothly.They often confused in ordered pairs whether first one is x-coordinate or y-ordinate.Also the results shows weakness ness in choosing quadrants. Many mistakes were due to their careless also .For example writing as an ordered pair from the table ,forgetting to put negative signs etc., In the next section we are going to see the possible causes for the mistakes and the remedial activity planned to help the students.

Table: 3 Possible Causes and Remedial Activity

S.No	**Topics**	**Mistakes in Detail**	**Possible causes**	**Planned remedial activity. (See ANNEXURE 2)**
	Mistakes when calculating x^2	1. Putting "-" sign when finding X^2.	Weakness in integer multiplication rules.	Finding squares worksheets.

1		2.Wrong calculation of X^2	Weakness in squaring integers.	Integer multiplication worksheets
2.	Mistakes when calculating bx	1.Sign mistakes -5(-2) = -10	Weakness in integer multiplication rules.	Integer multiplication worksheets
		2. Multiplying co-efficient of x with x^2 values instead of x	Careless mistake.	Calculation table worksheets
		3. Multiplying co-efficient of x with values of x Wrongly. Example: -3(-2) =9 5 x 0 =-5	Weakness in multiplication. Careless mistake.	
		4. Treating co-efficient of x as constant	Careless mistake.	
3.	Mistakes in values of y	Wrong calculation when adding values of $x^2 + bx + c$	Weakness in integer addition rules.	Calculation table worksheets
	Mistakes in ordered pairs	1.when writing as an ordered pair interchanging x and y values. 2.Missing the signs when writing as an ordered pair. 3. writing wrong set of values as an ordered pair.	1.Weakness in the ordered pair concept. 2.careless mistake	Writing as Ordered pair worksheets.

Table: 4 Possible Causes and Remedial Activity

S.No.	Topics	Mistakes in Detail	Possible causes	Planned remedial activity
1.		No curve: 1.Using ruler scale to join points.	Weakness in knowing the properties of parabola	Using geogebra to teach properties.

	Graph	2.Drawing curves not smoothly.	concept.	Draw graphs for given points worksheet.
2.	Scaling in Axes	Incorrect scaling: 1.Skipping one unit from origin in axes and marking x or y values.	Careless mistake	Scaling in axe worksheet
		2. Taking 1cm = 1-unit in one side of the axis and 1cm=2 units in the other side.	1.careless mistake. 2.Weakness in the concept of scaling	
3.	Plotting points	Incorrect plotting of points: 1.Confusion in Plotting order Pairs. Example: (-2,0) as (0, -2)	Weakness in plotting an ordered pair as a point.	Find the ordered pairs worksheet. Plotting points worksheet.
		2.Points were plotted in the wrong quadrants.	Weakness in the quadrants concept	
		3.points were plotted completely which are not in the table.	Careless mistake	

As seen in the Tables 3 and 4 possible causes for all the mistakes and were identify remedial activity were planned. Students were given enough drills on the activities for 20 days.

Using ICT and patterns in quadratic sequence

The general form of a quadratic function is f(x) =$ax^2 + bx + c$, where a, b, c are constants and a≠ 0.First they have to understand how the constants a ,b ,c affect the graph of the parabola .If they understand how 'a' affects the opening and width of the parabola ,'b' affects the location of the vertex with respect to the y-axis and 'c' affects the vertical shift .

So I decided to use Geogebra for that. I asked students to find out how the constants affect the graph by varying the values of the constants. Now they understood the properties of quadratic graph and can visualize the graph looking at the values and signs of a, b and c.

X	-5	-4	-3	-2	-1	0	1
Y	2	-3	-6	-7	-6	-3	2
1^{st} difference	2-(-3)=5	-3-(-6)=3	-6-(-7)=1	-7-(-6)=-1	-6-(-3)=-3	-3-(+2)=-5	
2^{nd} difference	5-3=2	3-1=2	1-(-1)=2	-1-(-3)=2	-3-(-5)=2		

Next the students were taught how to cross-check the calculation table and also the graph.

Any sequence having common second difference is a quadratic sequence. Also there will be some pattern in the first difference of a quadratic sequence as seen below.

$x^2 + 4x - 3$ If a = 1 and **b is even** the **pattern of first difference will be 1-3-5-7…**

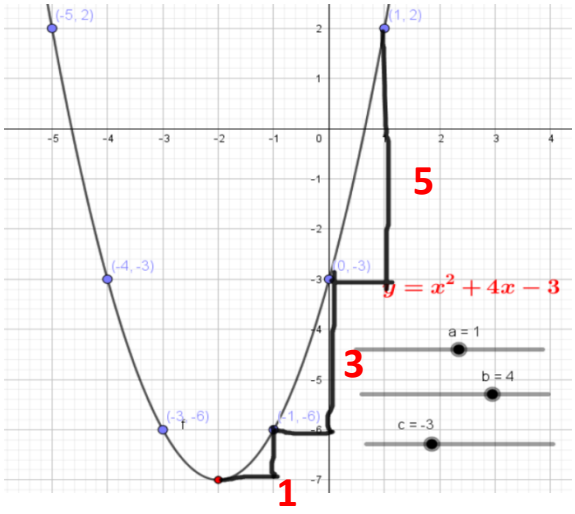


Figure:1 Pattern

These patterns not only help us to cross check the calculation table, but also helps to cross check the graph. After completing the graph, students can check whether the pattern is there or not. If the pattern is not there, then they can know there will be some mistakes in the calculation table or plotting of points.

$x^2 - 3x + 8$ If a = 1 and **b is odd** the **pattern of first difference will be 0-2-4-6…**

X	-2	-1	0	1	2	3	4
Y	18	12	8	6	6	8	12
1st difference	18-12=6	12-8=4	8-6=2	6-6=0	6-8=-2	8-12=-4	
2nd difference	6-4=2	4-2=2	2-0=2	0-(-2)=2	-2-(-4)=2		

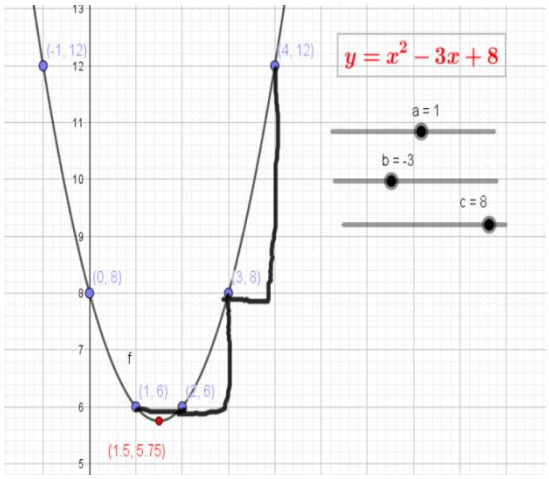


Figure:1 Pattern

In the first difference patterns, the sign changes a...5,3,1, -1, -3, -5 … shows' that after a point the direction of the graph changes. That point is called the vertex of the parabola. Same thing happens to 6,4,2,0, -2, -4, -6…also.

These patterns depend upon the co-efficient of x. If it is odd, the pattern will be even numbers. If it is even, the pattern will be odd numbers.

Points to remember:

Checking the following points will help the students to check the graph whether its correct or not.

y = $ax^2 + bx + c$ where a=1

1. If a is positive, the parabola opens upwards and if a is negative then the parabola opens downwards.
2. If b is 0, then the vertex will be on the y-axis.
3. If b is negative, then the vertex will be on the left side of y-axis.

If b is positive, then the vertex will be on the right side of y-axis.

- If b is even, the vertex point would be plotted in the graph as it is present in the calculation table. Also check, whether odd pattern is between the points.
- If b is odd, then the vertex point will not be in the graph as it is not in the calculation table. Also check whether even pattern is between the points.
- Check the symmetry of the graph and check the distance of the points on either side from the axis of symmetry.

The above points were taught to the students to check the graph after they finished. It will help them to find out the mistakes in the graph and hence mistakes in the calculation table.

Plotting points without the Calculation table

If the student knows how to find the vertex or axis of symmetry, then using the above said patterns' they can plot points even without the calculation table.

$y = a (x – h)^2 + k$ is the vertex form where (h, k) is the vertex.

To find the x coordinate h we can use the formula h= $\frac{-b}{2a}$ (or) change the sign of b and make into half. (if a=1)

To find the y coordinate, the actual formula is $\frac{-(b^2-4ac)}{4a}$.But it is very hard for them to find and time consuming also. so we kept 'a' and 'c' constant and varied the value of the co-effcient of x value in odd numbers well as even numbers and recorded the values as shown in the table.

Coefficient of x (b)	Number to be subtracted from constant (c)	Coefficient of x (b)	Number to be subtracted from constant (c)
1	0.25	0	0
3	.25	2	1
5	.25	4	4
7	2.25	6	9
9	0.25	8	16

We found out formulae by finding a pattern and the formula is given below.

k = (c-d)-0.25 $d = \left(\frac{b-1}{2}\right)\left(\frac{b+1}{2}\right) = (b^2 - 1)/4$ if b is odd.

k =c-d, where $d = \left(\frac{b}{2}\right)^2$ if b is even.

After finding the vertex they can plot by moving up according to the pattern on both sides and complete the parabola as seen previously in the **pattern figure 1** and **pattern figure 2.**

With all these remedial activities and patterns knowledge the students were prepared for the post test. The post test results were tabulated below.

Table: 5 Mistakes in the Calculation Table-Post Test

S.No	Topics	Mistakes	Frequency	% of students did
1	Mistakes when calculating x^2	1. Putting "-" sign when finding X^2. Examples: $(-3)^2 = -9$ $(-2)^2 = -4$	1	2%
		2.Wrong calculation of X^2 Examples: $(3)^2 = 6$ $(-4)^2 = 8$	2	3%
2.	Mistakes when calculating bx	1.Sign mistakes -5(-2) = -10 3(-2) = 6	3	5%
		3. Multiplying co-efficient of x with values of x Wrongly. Examples -3(-2) =9 5 x 0 =-5	4	7%
		4. Treating co-efficient of x as constant For -5x writing -5 in the entire row.	1	2%
3.	Mistakes in values of y	Wrong calculation when adding values of $x^2 + bx + c$ Example: 4-10-6=12	8	13%

4.	Mistakes in ordered pairs	1.when writing as an ordered pair interchanging x and y values. 2.Missing the signs when writing as an ordered pair. 3. writing wrong set of values as an ordered pair.	3	5%

Table: 6 Mistakes in The Graph-Post Test

S.No	Topics	Mistakes in Detail	Frequency	% of students did
1.	Graph	2. Drawing curves not smoothly.	6	10%
2.	Scaling in Axes	2. Taking 1cm = 1-unit in one side of the axis and 1cm=2 units in the other side.	1	2%
3.	Plotting points	Incorrect plotting of points: 1. Confusion in Plotting order Pairs. Example: (-2,0) as (0, -2)	3	5%
		2. Points were plotted in the wrong quadrants.	5	8%
		3. Points were plotted completely which are not in the table.	3	5%

Conclusion

Table: 7 Mistakes In The Calculation Table-Comparison

S.No.	Topics	Mistakes	Pre-test	Post-test
1	Mistakes when calculating x^2	1. Putting "-" sign when finding X^2. Examples: $(-3)^2 = -9$ $(-2)^2 = -4$	17%	2%
		2.Wrong calculation of X^2 Examples: $(3)^2 = 6$ $(-4)^2 = 8$	15%	3%
2.	Mistakes when calculating bx	1.Sign mistakes -5(-2) = -10 3(-2) = 6	15%	5%
		2. Multiplying co-efficient of x with x^2 values instead of x	17%	--
		3. Multiplying co-efficient of x with values of x Wrongly. Example: -3(-2) =9 5 x 0 =-5	17%	7%
		4. Treating co-efficient of x as constant For -5x writing -5 in the entire row.	12%	2%
3.	Mistakes in values of y	Wrong calculation when adding values of $x^2 + bx + c$ Example: 4-10-6=12	30%	13%
4.	Mistakes in ordered pairs	1.when writing as an ordered pair interchanging x and y values. 2.Missing the signs when writing as an ordered pair. 3. writing wrong set of values as an ordered pair.	18%	5%

Table: 8 Mistakes In The Graph-Comparision

S.No	Topics	Mistakes	Pre-test	Post-test
1.	Graph	No curve: 1. Using ruler scale to join points.	5%	--
		2. Drawing curves not smoothly.	25%	10%
2.	Scaling in Axes	Incorrect scaling: 1. Skipping one unit from origin or axes and marking x or y values.	3%	--
		2. Taking 1cm = 1-unit on one side of the axis and 1cm=2 units in the other side.	7%	2%
3.	Plotting points	Incorrect plotting of points: 1. Confusion in Plotting ordered Pairs. Example: (-2,0) as (0, -2)	12%	5%
		2. Points were plotted in the wrong quadrants.	20%	8%
		3. Points were plotted which are not in the table.	13%	5%

As we have seen in the Tables 7 and 8, it is evident that after the remedial activities implemented using drill worksheets, Geogebra (ICT) and patterns in quadratic sequence the percentage of students doing mistakes were considerably reduced.

Some of the mistakes were not done by any students. But it is noted that a fair amount of students are still weak in integer multiplication and addition. Also, still some students find difficulty to draw the curve smoothly. Careless mistakes also play a major role in the achievement process.

If we teach patterns in quadratic sequence, vertex and axis of symmetry it will definitely help the students to verify the graph whether it is correct or not. Thus they can cross check and correct the mistakes in the calculation table also.

Finally, the drill worksheets on integer multiplication and addition, Geogebra and patterns knowledge in quadratic sequence helped students to minimize their mistakes in graphing quadratic functions

Implications

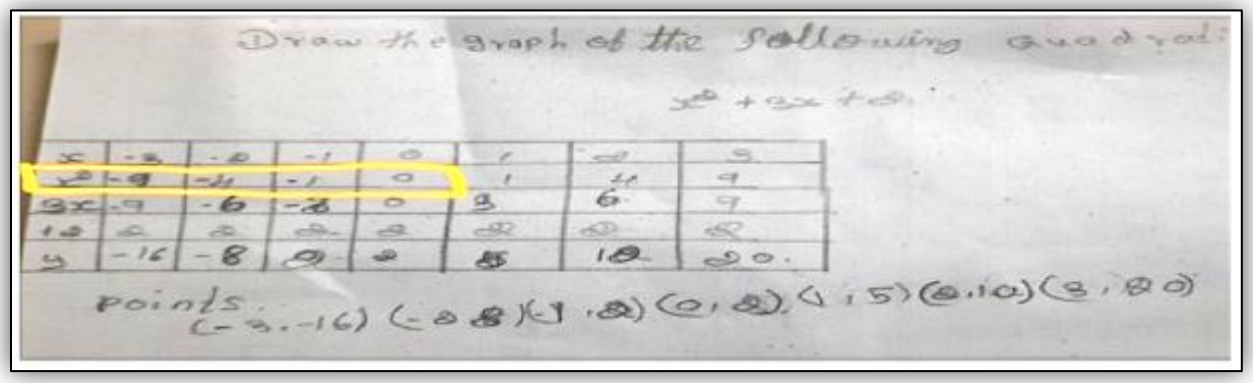
This study clearly shows that the Mathematics teachers should definitely concentrate on basic skills in the upper primary stage. Because the students are still finding difficult in integers even in class 10, using ICT effectively is also need of the hour.

Students can be encouraged to find what will be the patterns if a=2 in the quadratic function $y = ax^2 + bx + c$. Such activities will make the students appreciate the beauty of mathematics and create interest towards Mathematics.

Annexure

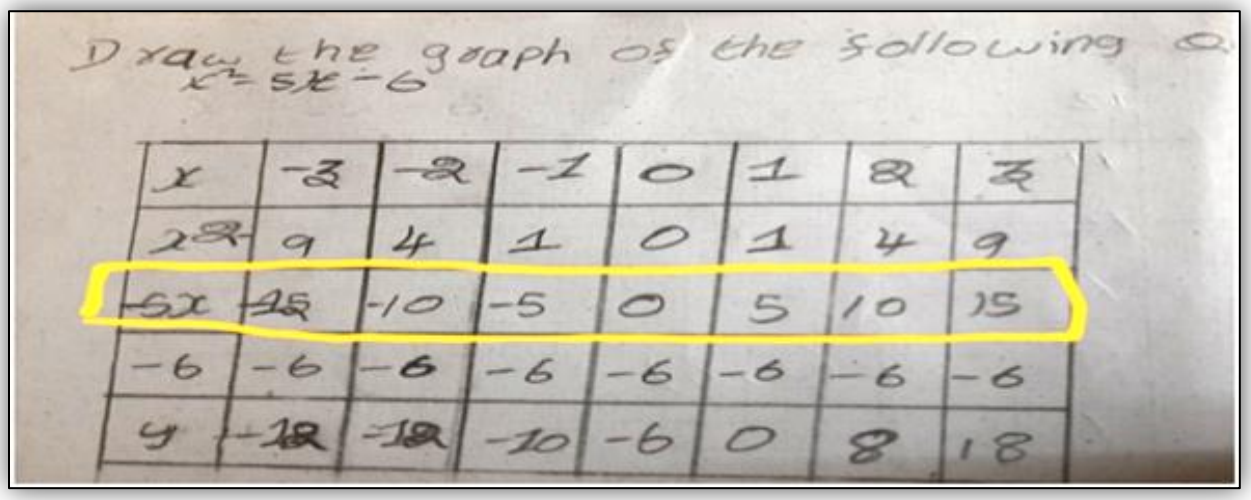
1. **Mistakes in the calculation table**

"-" Sign in the values of X^2 **Mistakes when Finding X^2**

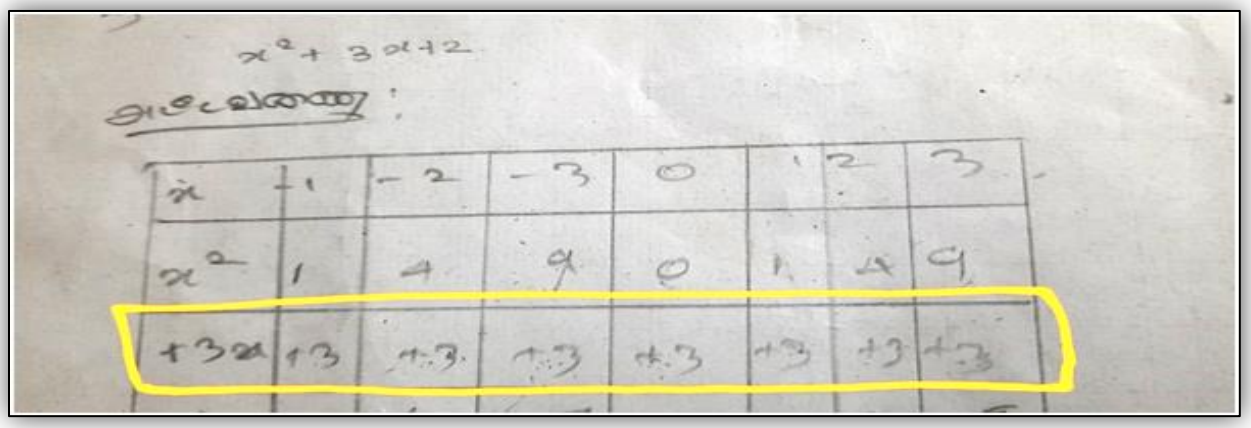


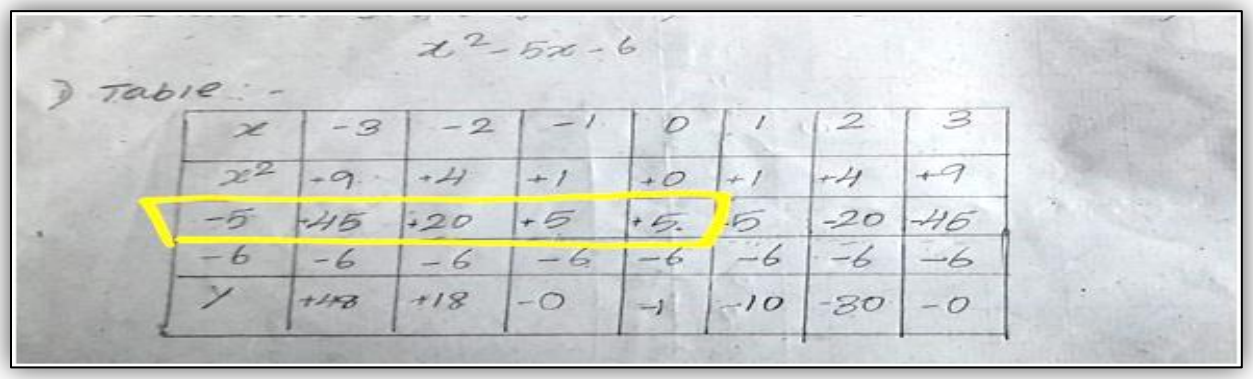
Wrong Sign When Substituting Values Of X

$$-5(-1) = 5$$

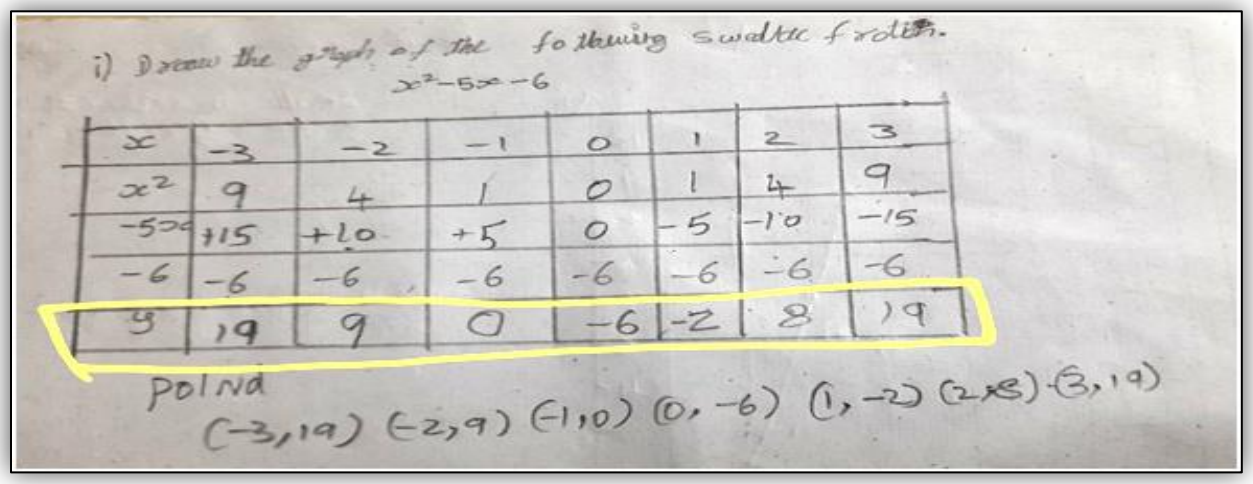


Treating co-efficient of x as constant

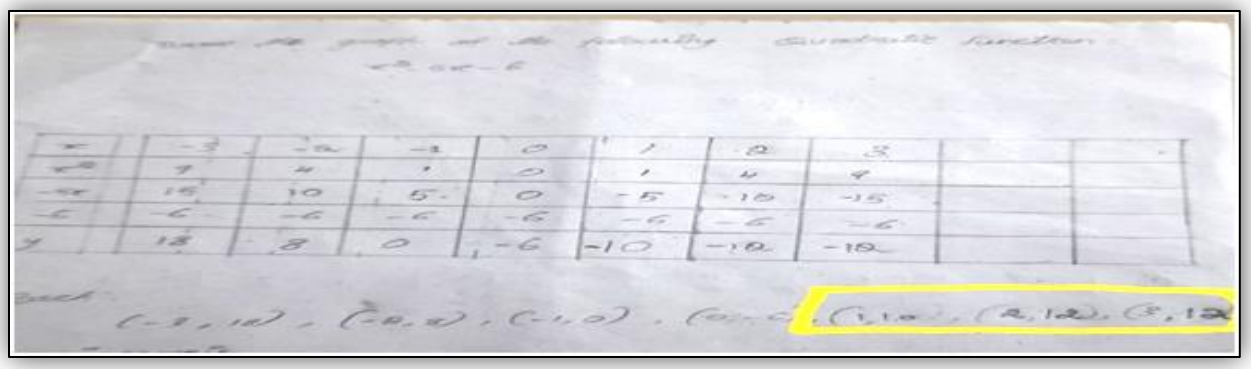




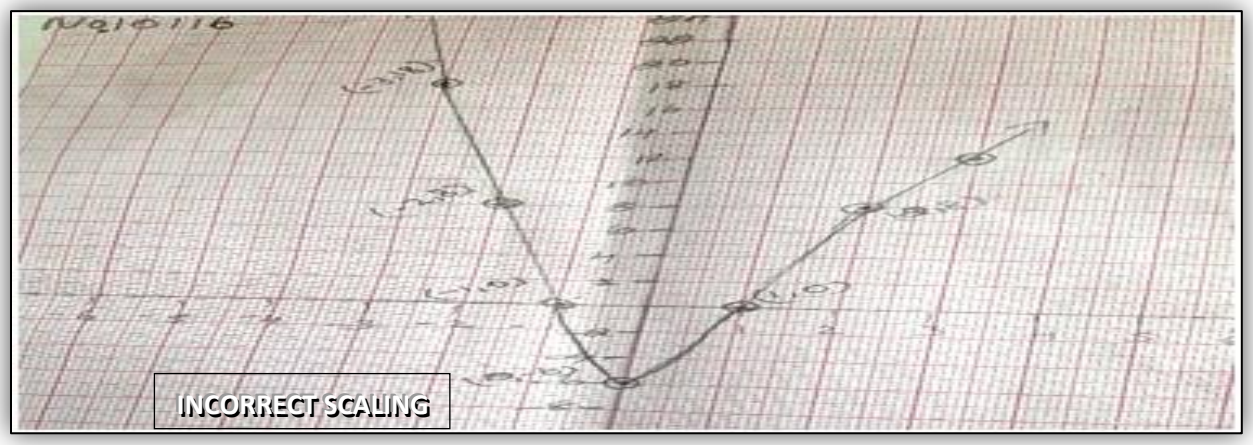
Mistakes when calculating the y-values

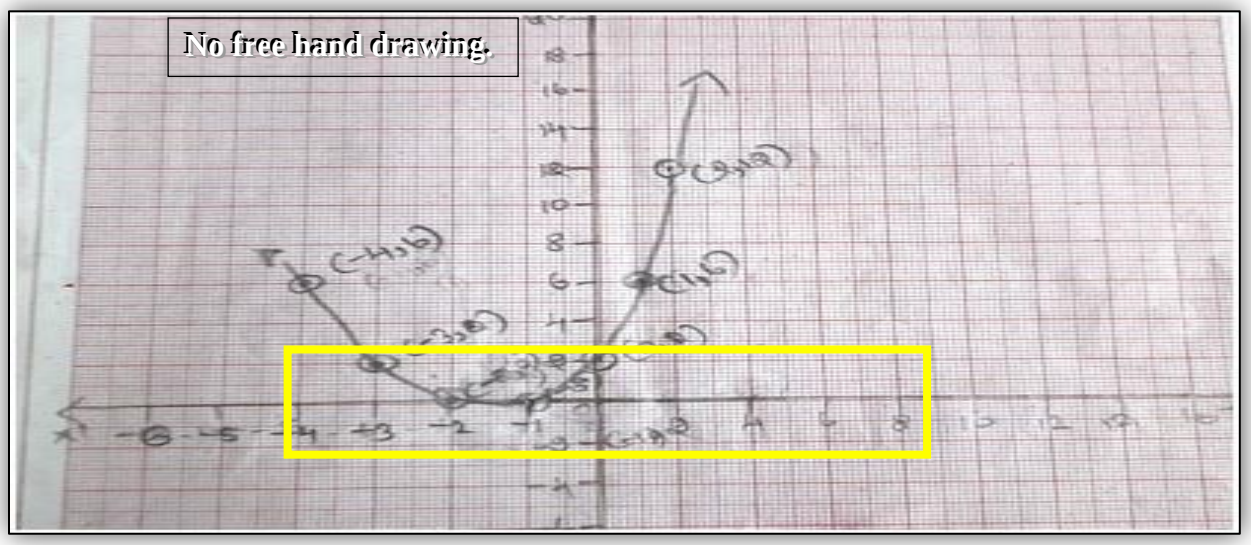


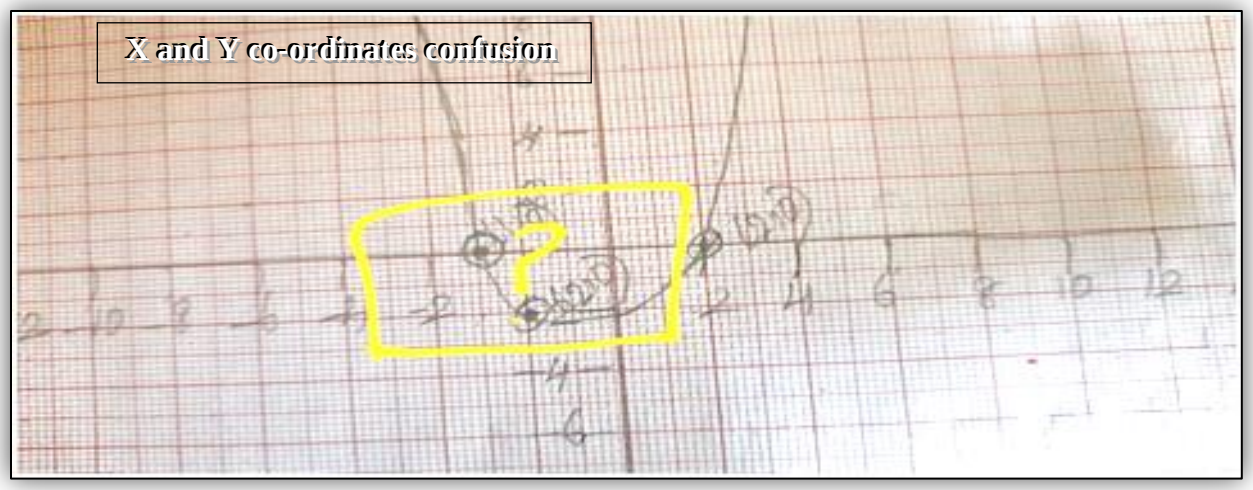
Mistakes when writing as ordered pair

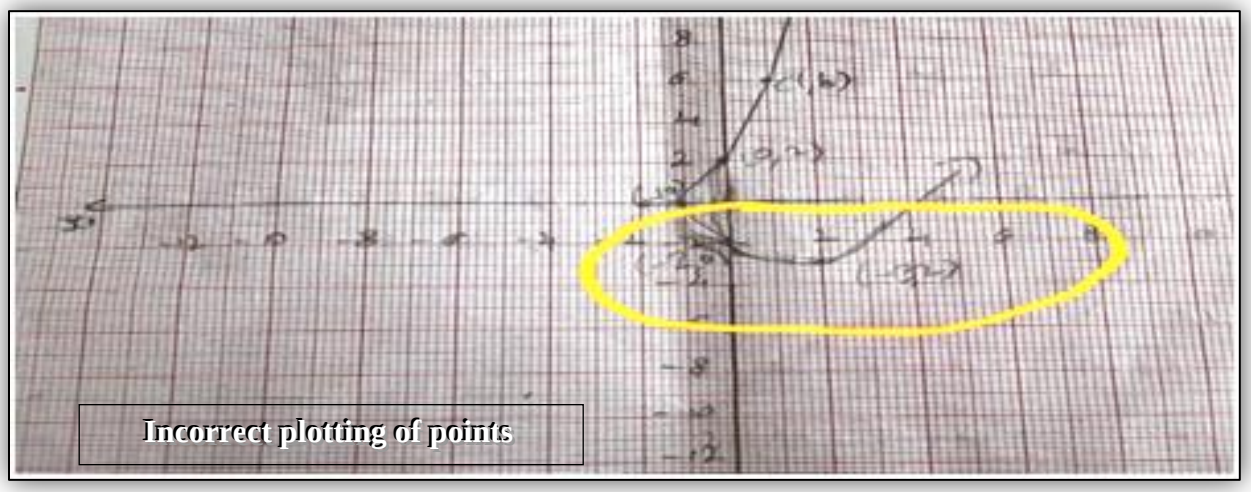


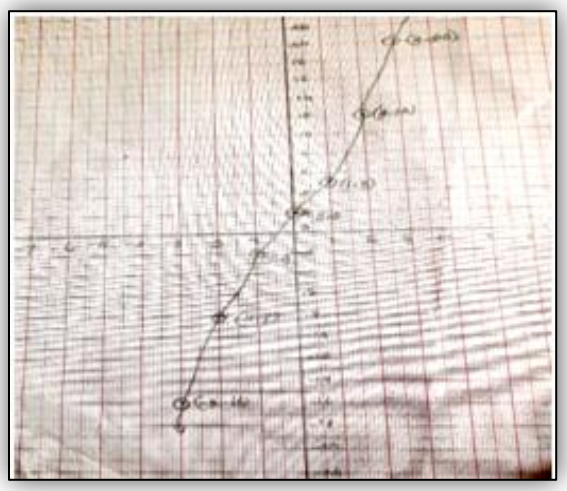
2. Mistakes in the graph:

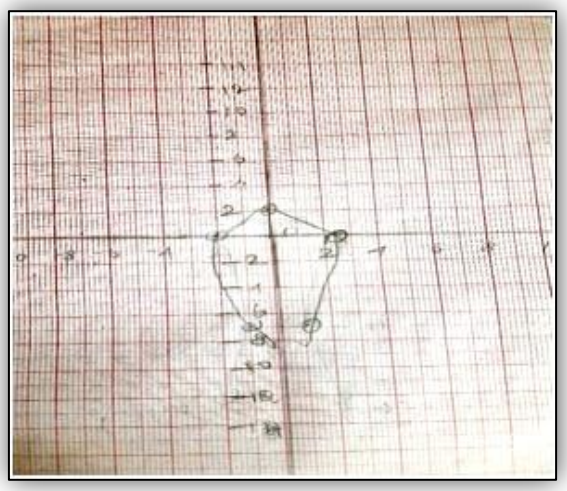




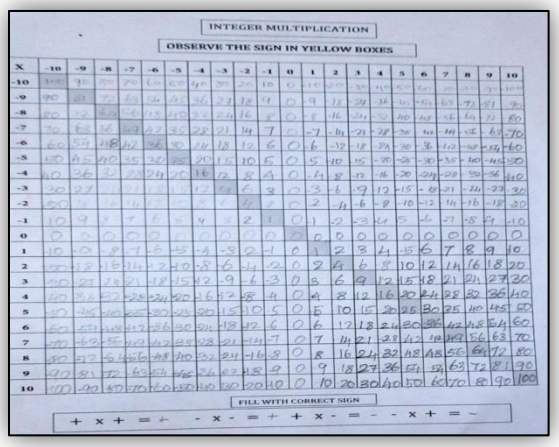

X and Y co-ordinates confusion

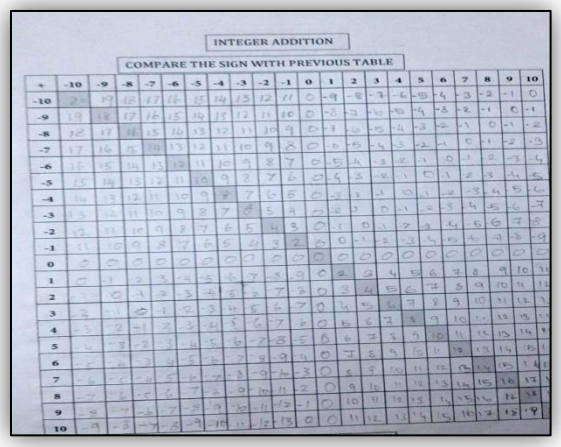

Incorrect plotting of points

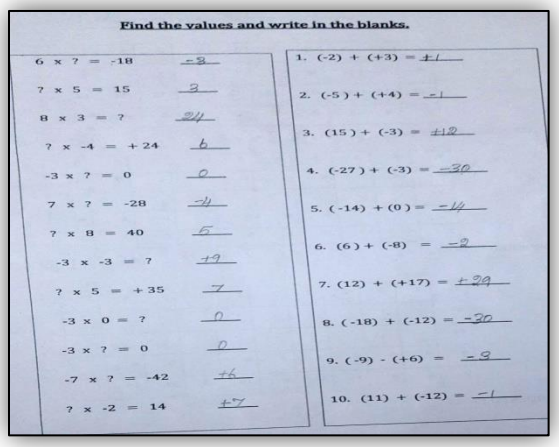


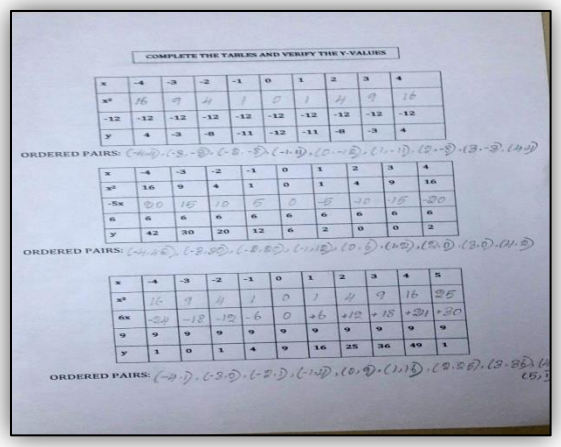


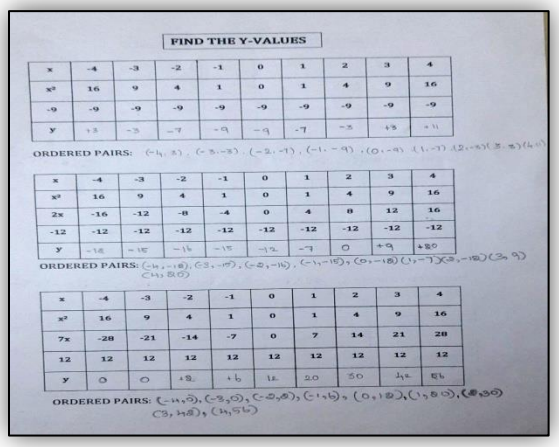
Annexure 2: Remedial Worksheets

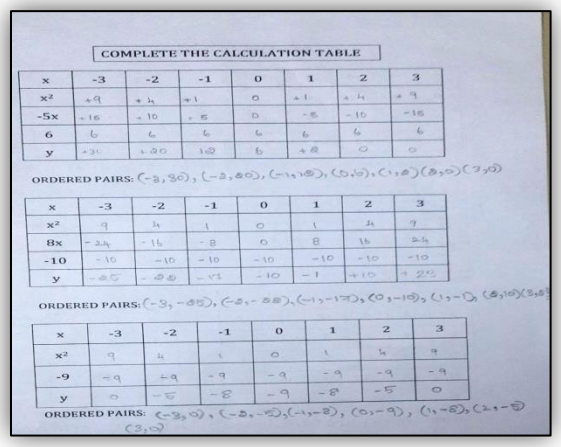


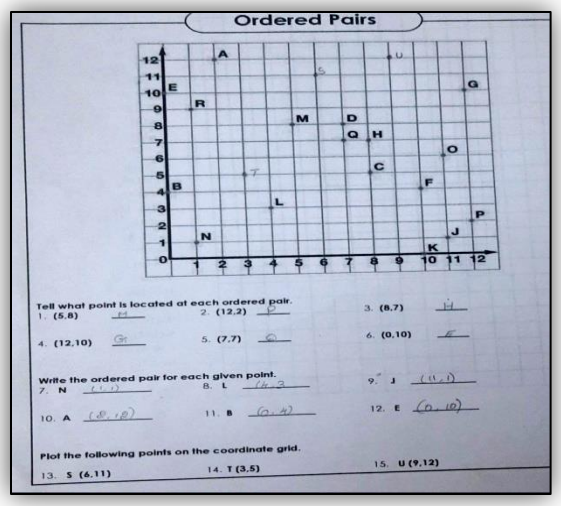


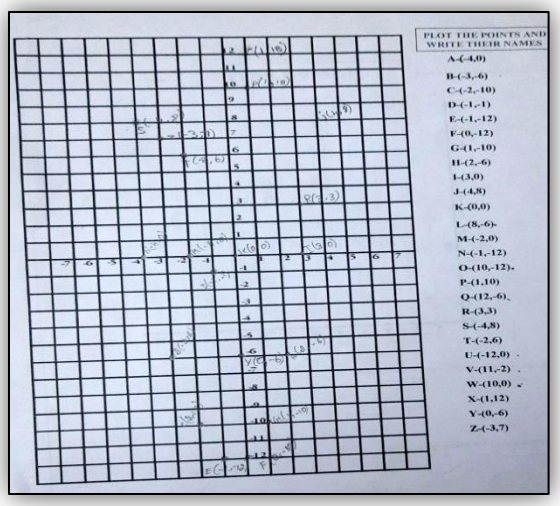


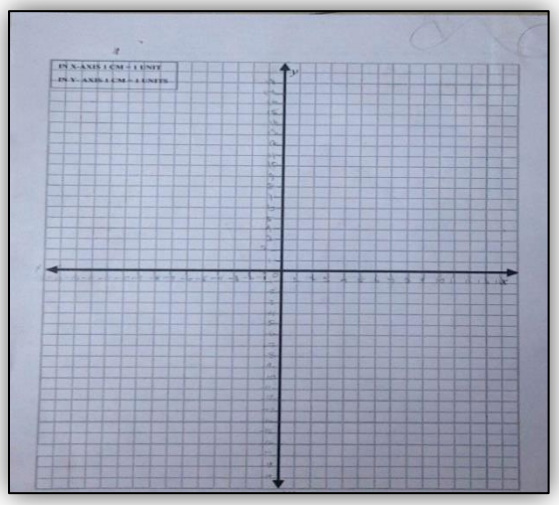


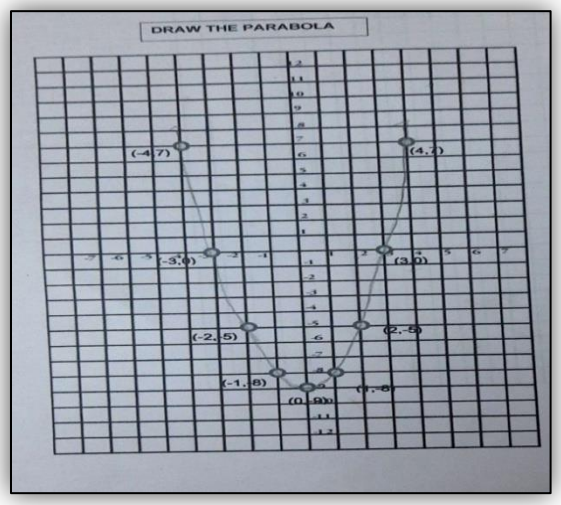


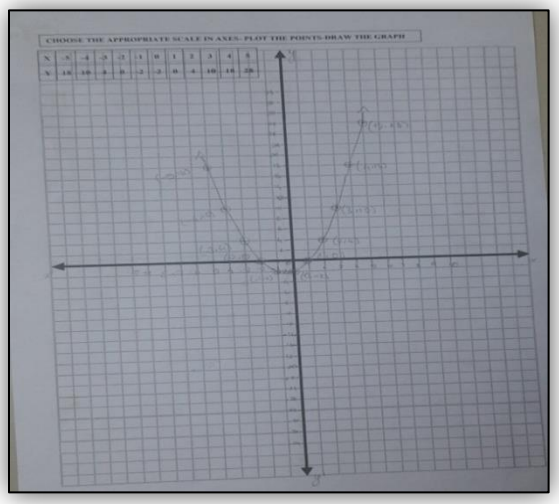


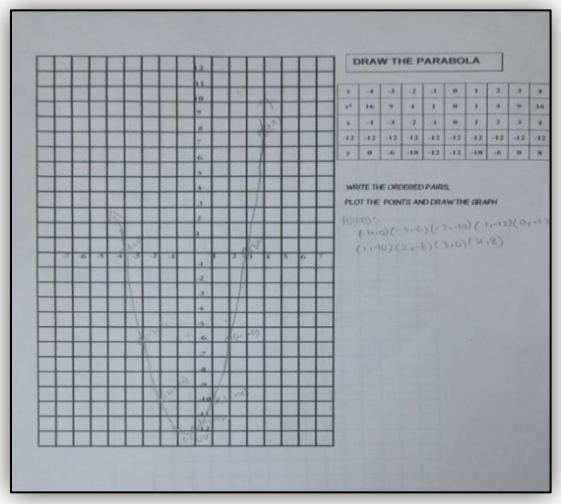

Ordered Pairs
Tell what point is located at each ordered pair.
1. (5,8) M
2. (12,2) P
3. (8,7) H
4. (12,10) G
5. (7,7) Q
6. (0,10) E
Write the ordered pair for each given point.
7. N (1,1)
8. L (4,3)
9. J (11,1)
10. A (8,12)
11. B (0,4)
12. E (0,10)
Plot the following points on the coordinate grid.
13. S (6,11)
14. T (3,5)
15. U (9,12)


PLOT THE POINTS AND WRITE THEIR NAMES
A-(-4,0)
B-(-3,-6)
C-(-2,-10)
D-(-1,-1)
E-(-1,-12)
F-(0,-12)
G-(1,-10)
H-(2,-6)
I-(3,0)
J-(4,8)
K-(0,0)
L-(8,-6)-
M-(-2,0)
N-(-1,-12)
O-(10,-12)-
P-(1,10)
Q-(12,-6),
R-(3,3)
S-(-4,8)
T-(-2,6)
U-(-12,0)
V-(11,-2)
W-(10,0)
X-(1,12)
Y-(0,-6)
Z-(-3,7)


IN X-AXIS 1 CM = 1 UNIT
IN Y-AXIS 1 CM = 1 UNIT


DRAW THE PARABOLA
(-4,7)
(4,2)
(-3,0)
(3,0)
(-2,5)
(2,-5)
(-1,8)
(1,-8)
(0,9)


CHOOSE THE APPROPRIATE SCALE IN AXES- PLOT THE POINTS-DRAW THE GRAPH


DRAW THE PARABOLA
WRITE THE ORDERED PAIRS.
PLOT THE POINTS AND DRAW THE GRAPH

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