Continuous Professional Development in Mathematics: Higher Order Thinking Skills

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Abstract- *Teaching high order thinking skills (HOTS) is currently at the centre of educational attention. In general, measures of high order thinking include all intellectual tasks that call for more than the retrieval of information. Five fundamental HOTS have been identified in Mathematics. They are: problem solving skills, inquiring skills, reasoning skills, communicating skills and conceptualizing skills. These fundamental and intertwining ways of learning mathematics, thinking and using mathematical knowledge are considered important in mathematics education. In fact, many of students' problems in learning mathematics originate from their weaknesses in one or more of these skills. Students are expected to enhance the development of these skills and use them to construct their mathematical knowledge, and hence engage in life-long learning.*

Keywords: HOTS, CPD in Mathematics, Bloom's Taxonomy.

Introduction

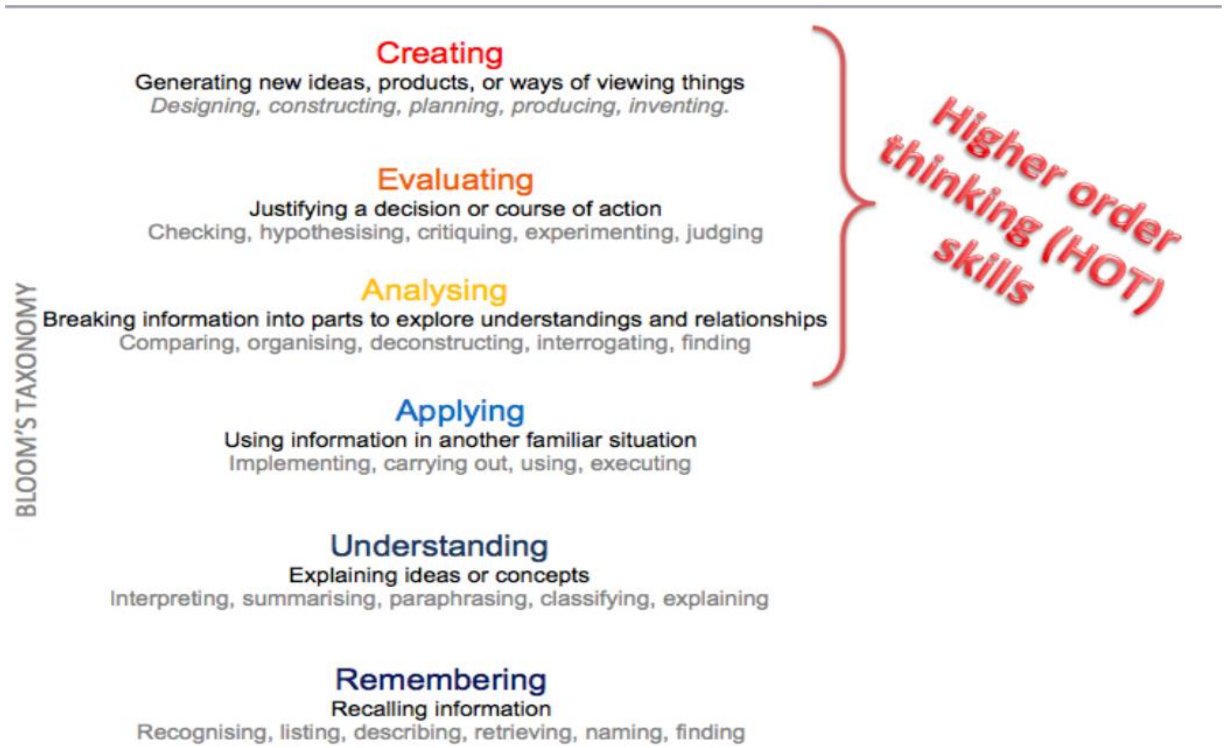
Teachers should note the following points. First, there is no simple, clear and universally accepted definition of HOTS. In fact, they may be arranged into several overlapping categories such as metacognitive skills, critical and creative thinking. Nevertheless, it is generally agreed that high order thinking is non-algorithmic and complex; it involves self-regulation of the thinking process and often yields multiple solutions to tasks. This generally agreed features of HOTS give rise to the common technique of posing open-ended problems for fostering HOTS in mathematics classes. This technique gives students chances to think about mathematics and to talk about mathematics with each other and with their teacher.

Second, these HOTS cannot be easily isolated from each other in mathematical work. For example, a student using reasoning skills to solve a problem may also be considered as demonstrating his/her problem solving skills. Similarly, communicating skills are always involved in doing mathematical tasks, and conceptualizing skills are engaged in all exploratory work.

Third, HOTS can be taught in isolation from specific contents, but incorporating them into content areas seems to be a popular way of teaching these skills. The key to generalizing HOTS for the student into a content area is to focus on a skill that was learned in one setting and to see its relevance in another.

Fourth, computer provides an excellent tool for teaching HOTS because of its interactive capabilities and its ability to present and stimulate problems. The computer is an effective medium to help teaching the skills initially and to promote generalizations. Using a computer enables students to spend more time on the thinking of tasks rather than on peripheral and trivial activities. It indirectly enhances thinking skills by making it possible for students to spend their overall time more effectively.

The backbone for all of this is Bloom's Taxonomy



Thus, Higher order skills are skills involving analysis, evaluation and synthesis (creation of new knowledge). These are thought to be of a 'higher order', requiring different learning and teaching methods than the learning of facts and concepts.

Also, Higher order thinking involves the learning of complex judgmental skills such as critical thinking and problem solving.

Higher order thinking is more difficult to learn or teach but also more valuable because such skills are more likely to be usable in new and unfamiliar situations.

And, Higher order questions require answers that go beyond simple information and as such both the language and thinking behind them is more complex. They take learners into abstract language functions, such as giving and justifying opinions, speculation and hypothesising.

Before going to discuss about Bloom's Taxonomies, let's discuss about skills in little detail here.

Problem Solving Skills

Problem solving is an integral part of all mathematics learning and it involves identifying obstacles, constraints or unexpected patterns, trying different procedures and evaluating or justifying the solution. The National Council of Teachers of Mathematics (NCTM) considers problem solving as a process of applying previously acquired knowledge to new and unfamiliar (or unforeseen) situations. To solve a problem, students draw on their knowledge and develop new mathematical understandings. They should also acquire ways of thinking, develop confidence and habits of persistence in unfamiliar situations through the problem solving process.

The general problem solving strategies embrace understanding the problem, devising a plan of solving the problem, carrying out the plan, examining the reasonableness of the result and making evaluation. These four phases have formed a framework for problem solving in many mathematics textbooks.

Successful problem solving involves the process of coordinating previous mathematical knowledge and experience to develop a solution of a problem for which a procedure for determining the solution is not known. Intuition may also be involved in the thinking process. Therefore, in the problem solving process, students may make conjectures and try many different ways to tackle the problem. Teachers should note that any method, which can be properly used to solve a problem, is a “correct” method. Teachers should not discourage a student merely because his/her method is too long or too complicated. Instead, teachers should guide the student with patience to examine the process or method he/she adopts.

Both non-routine and open-ended problems give students more opportunities to demonstrate their problem solving skill. However, teachers should note that problems of the similar type, even non-routine or open-ended problems, when done repeatedly, will become routine problems and hence lose their function of fostering students’ HOTS.

Teachers should note that problem solving is more than solving problems. The latter involves strategy of writing down the rule (or formula), demonstrating how to use the rule and providing exercises to students for practising the rule. The former, on the other hand, emphasizes on heuristic processes, developing flexibility and creativity in applying mathematical ideas and skills to unfamiliar questions. Students can acquire opportunities to develop their interest in mathematics and foster their capability of independent thinking through problem solving.

Inquiring Skills

Inquiring involves discovering or constructing knowledge through questioning or testing a hypothesis. Observation, analysis, summarizing and verification are the essential elements in carrying out inquiring activities. Inquiring activities mainly involve self-learning processes, but suitable guidance from teachers is sometimes necessary depending on the abilities of students and the complexity of the activities. Posing questions is one popularly adopted means to guide students to make exploration. In fact, well-designed questions are useful to stimulate students to discover similarities, differences, patterns and trends. Students may also be asked to test mathematical conjectures, which enable them to participate in a more active role in the learning process.

Designed inquiring activities should cope with the abilities of students so that they can enjoy the discovery of mathematical results. Moreover, it may be more effective to arrange students in small groups (whenever possible) because it is easier for them to put forward their ideas. The following list of verbs may be helpful in guiding students to perform inquiring activities: explore, discover, create, prove, validate, construct, predict, experiment, investigate, etc.

Inquiring activities usually require some teaching aids. Teachers should therefore make proper preparations well before the lesson so that adequate sets of aids are available. The following questions should be considered before organizing the inquiring activities in class:

- Will students be grouped when performing the activities? If yes, how many groups should be organized?
- How can we ensure that the right amount of guidance (in the form of hints or questions) is provided? (It should be noted that either insufficient or too much guidance will do no good to students.)
- When computers are available, what software could be used? Is there sufficient software for the whole class? If no, what can be done?

Communicating Skills

Communication involves receiving and sharing ideas and can be expressed in the forms of numbers, symbols, diagrams, graphs, charts, models and simulations. It is viewed as an integral part of mathematics instruction as it helps clarify concepts and builds meaning for ideas. Through the communication process, students learn to be clear and convincing in presenting their mathematical ideas, which definitely helps develop their logical thinking.

Since mathematics is very often conveyed in symbols, oral and written communication about mathematical ideas is often overlooked by teachers. However, it should be noted that both oral and written languages are needed to describe, explain and justify mathematical ideas. These abilities can help students to clarify their thinking and sharpen their understanding of concepts and procedures. Furthermore, during the process of communicating, students may construct, refine and consolidate their mathematical understandings.

Among all forms, written communication is of special importance because it provides students with a record of their own thinking and ideas. Moreover, the process of writing in mathematics learning promotes students' active involvement. Nevertheless, students should be reminded to write concisely, precisely and neatly as mathematics needs clear, consistent, concise and cogent language.

Communication can be fostered in many ways. For example, students can be asked to describe a practical task and to tell what characteristics they discover. Investigative activities and project work are ideal tasks for developing students' communicating skill. Open-ended questions that allow students to construct their own responses and encourage divergent or creative thinking furnish fertile areas for communication. Small group discussions and debates are also helpful. They can be used to encourage students to read, write and discuss their mathematical ideas. However, teachers should pay attention to the suitable arrangement of classroom and groupings for facilitating students to share their ideas. Small collaborative groups afford opportunities to explore ideas while whole-class discussions can be used to compare and contrast ideas from individual students.

Reasoning Skills

Reasoning is drawing conclusions from evidence, grounds or assumptions. It involves developing logical arguments to deduce or infer conclusions. Reasoning may be classified into inductive reasoning and deductive reasoning. Inductive reasoning works from specific observations to broader generalizations and theories while deductive reasoning moves from the other way round, that is, from the more general to the more specific. By its very nature, the inductive reasoning method is more open-ended and exploratory and the deductive one is narrower in nature and is usually concerned with testing or verifying hypotheses and theories. Therefore, finding the general term of a sequence like 1, 3, 5, 7, 9, …… involves inductive reasoning while doing a geometric proof by applying a geometrical theorem (say, the corresponding angles of two similar triangles are equal) involves deductive reasoning.

Since reasoning is a fundamental aspect of mathematics, being able to reason is essential to the understanding of mathematical concepts. By making investigations and conjectures, developing and evaluating mathematical arguments, justifying results, etc., students are able to understand and appreciate the power of reasoning and produce proofs, which entail logical deductions of conclusions from theories and hypotheses. Reasoning, like other HOTS, cannot be taught in a single lesson. Instead, it is a habit of mind and should be a consistent part of students' mathematical experience. It is fostered or developed through a prolonged learning of mathematics in different contexts.

To develop reasoning skills, students should be familiar with the following:

1. Sorting and classifying information, interpreting information and presenting results with pictures, diagrams, graphs, models, symbols and tables.
2. Describing, generalising, justifying patterns presented in a variety of forms and contexts, making conjectures, thinking flexibly, proving and refuting, recognising logical and illogical arguments, following a chain of reasoning, making deductions and demonstrating methods of mathematical proof (including proof by contradiction, counter-example and induction).

It should be noted that reasoning involves informal thinking, making hypotheses and validating them. Students should be encouraged to justify their answers and solution processes. Questions which are useful to help students develop their reasoning skills include:

- Why do you think it is true/false?
- If we go in this way, what happens? How do you know?
- If we change angle A to 90 degrees, will the result remain the same?
- If the lines are not parallel, the theorem is not true. Why?
- Pythagoras' theorem is true for any triangle. Comment.

Conceptualizing Skills

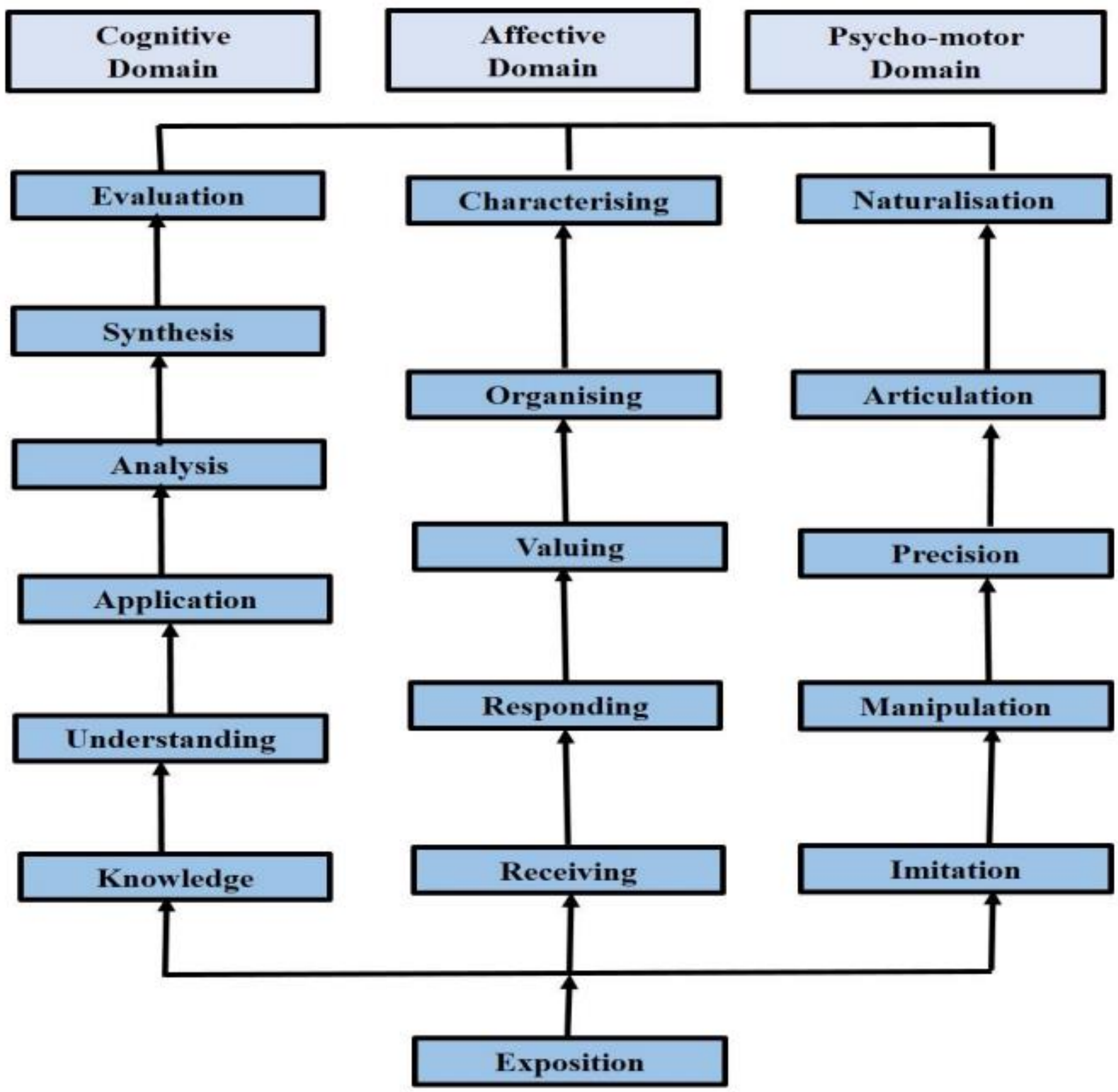
Conceptualizing involves organizing and reorganizing of knowledge through perceiving and thinking about particular experiences in order to abstract patterns and ideas and generalize from the particular experiences. The formation of concepts involves classifying and abstracting of previous experiences.

The particular problem of mathematics lies in its abstractness and generality. Abstract concepts cannot be communicated to students by a definition but only by arranging for him/her to encounter a suitable collection of examples. It follows that abstract concepts should be backed up by an abundance of mathematical and daily-life examples. Teachers need to provide students with a clear guidance to construct mathematical concepts from the examples and use these concepts to solve problems in unfamiliar situations.

When a new concept is introduced (like "All symmetrical triangles are similar."), examples or counter-examples may be provided for illustration. Students may also be asked to explore the information relevant to the concept (like similar triangles or symmetrical triangles) and classify the similarities and differences in the examples.

In short, to help students build up mathematical concepts, suitable examples or activities, which allow students to construct new concepts independently, are necessary.

The following graphics inculcating different domains may be referred here:



Interrelation between Different Domains

The tripartite division of instructional objectives into three domains is not an exclusive one. Firstly, the achievement in one domain is to a quite high degree dependent on the learner's status in others. For instance, understanding (comprehension) may be a prerequisite for attaching proper value to an object or proper cognition necessary for arousing proper interest. Similarly, interests and attitudes affect the quality of performance in both cognitive and psycho-motor domains. Comprehension is a natural component of the precision level in the psycho-motor domain and similarly interests can be traced as affective components of almost all the cognitive proficiencies. Thus, there is some degree of parallelism and relationship between various objectives. Lower levels of each domain draw relatively closer to each other, e.g. knowing, receiving and imitating are very much interdependent among themselves. In the higher objectives too, there is perceivable parallelism. A particular object of one domain can correspond to one or more of objectives of the other domain. The implications of the nature of interrelationship of objectives are many. For example, while planning the instruction for an objective, it may be considered how best the relevant objective of other domains can be exploited, and developed. The affective domain has suffered on account of the over emphasis given to the objectives of cognitive domain such as "critical thinking" and "problem solving" without caring for emotive reaction arising out of such mental exercises. Even psycho-motor domain is also not properly dealt within the educational process. Children's growth in respect of the three domains, as a result of the learning experiences provided, is simultaneous. But every learning experience does not result in the same quantum of growth in all the three domains. Thus, when a learning experience is planned and provided, the pace of growth in different domains finally settles down at different levels in the different domains.

Developing Mathematics Thinking with HOTS Questions helpful in

To promote problem solving…

- What do you need to find out?
- What information do you have?
- What strategies are you going to use?
- Will you do it mentally? Will you do it with pencil and paper or a number line?
- Will a calculator help?
- What tools will you need?
- What do you think the answer or result will be?

To help when students get stuck …

- How would you describe the problem in your own words?
- What do you know what is not stated in the problem?
- What facts do you have?
- How did you tackle similar problems?
- Could you try it with simpler number? Fewer numbers? Using a number line?
- What about putting things in order?
- Would it help to create a diagram? Make a table? Draw a picture?
- Can you guess and check?

- Have you compared your work with anyone else? What did other members of your group try?

To make connections among ideas and applications …

- How does this relate to…?
- What ideas learned before were useful in solving this problem?
- What uses of mathematics did you find in the newspaper last night?
- Can you give me an example of…?

To encourage reflection …

- How did you get your answer?
- Does your answer seem reasonable? Why or why not?
- Can you describe your method to us all? Can you explain how it works?
- What if you had started with _____ rather than _____?
- What if you could only use…?
- What have you learned or found out today?
- Did you use or learn any new words today? What do they mean? How do you spell them?
- What are the key points or big ideas in this lesson?
- To help students build confidence and rely on their own understanding, ask…
- Why is that true?
- How did you reach that conclusion?
- Does that make sense?
- Can you make a model to show that?
- To help students learn to reason mathematically, ask…
- Is that true for all cases? Explain
- Can you think of a counter example?
- How would you prove that?
- What assumptions are you making?

To check student progress …

- Can you explain what your have done so far? What else is there to do?
- Why did you decide to use this method?
- Can you think of another method that might have worked?
- Is there a more efficient strategy?
- What do you notice when…?
- Why did you decide to organize your results like that?
- Do you think this would work with other numbers?
- Have you thought of all the possibilities? How can you be sure?

To help students collectively make sense of mathematics …

- What do you think about what _________ said?
- Do you agree? Why or why not?
- Does anyone have the same answer, but a different way to explain it?

- Do you understand what ________ is saying?
- Can you convince the rest of us that your answer makes sense?

To encourage conjecturing …

- What would happen if…? What if not?
- Do you see a pattern? Can you explain the pattern?
- What are some possibilities here?
- Can you predict the next one? What about the last one?
- What decision do you think he /she should make?

I. **Remember (Knowledge)** (Shallow processing: drawing out factual answers, testing recall and recognition)

Verbs for Objectives	Model Questions	Instructional Strategies
Choose	Who?	Highlighting
Describe	Where?	Rehearsal
Define	Which One?	Memorizing
Identify	What?	Mnemonics
Label	How?	
List	What is the best one?	
Locate	Why?	
Match	How much?	
Memorize	When?	
Name	What does it mean?	
Omit		
Recite		
Recognize		
Select		
State		

II. **Understand (Comprehension)** (translating, interpreting and extrapolating)

Verbs for Objectives	Model Questions	Instructional Strategies
Classify	State in your own words.	Key examples
Defend	Which are facts?	Emphasize connections
Demonstrate	What does this mean?	Elaborate concepts
Distinguish	Is this the same as. . .?	Summarize
Explain	Give an example.	Paraphrase
Express	Select the best definition.	STUDENTS explain
Extend	Condense this paragraph.	STUDENTS state the rule
Give example	What would happen if . . .?	"Why does this example. . .?"
Illustrate	State in one word . . .	Create visual representations
Indicate	Explain what is happening.	(concept maps, outlines, flow
Interrelate	What part doesn't fit?	charts organizers, analogies,
Interpret	Explain what is meant.	pro/con grids) PRO| CON
Infer	What expectations are there?	NOTE: The faculty member can

		show them, but they have to do it.
Match	What are they saying?	Metaphors, rubrics, heuristics
Paraphrase	This represents. . .	
Represent	What seems to be . . .?	
Restate	Is it valid that . . .?	
Rewrite	What seems likely?	
Select	Show in a graph, table.	
Show	Which statements support..?	
	What restrictions would you	
Summarize	add?	
Tell		
Translate		

III. **Apply** (Knowing when to apply; why to apply; and recognizing patterns of transfer to situations that are new, unfamiliar or have a new slant for students)

Verbs for Objectives	**Model Questions**	**Instructional Strategies**
Apply	Predict what would happen if	Modeling
	Choose the best statements	
Choose	That	Cognitive apprenticeships
Dramatize	Apply	"Mindful" practice – NOT just a
Explain	Judge the effects	"routine" practice
Generalize	What would result	Part and whole sequencing
Judge	Tell what would happen	Authentic situations
Organize	Tell how, when, where, why	"Coached" practice
Paint	Tell how much change there	Case studies
Prepare	Would be	Simulations
Produce	Identify the results of	Algorithms
Select		
Show		
Sketch		
Solve		
Use		

IV. **Analyze** (breaking down into parts, forms)

Verbs for Objectives	**Model Questions**	**Instructional Strategies**
Analyze	What is the function of . . .?	Models of thinking
Categorize	What's fact? Opinion?	Challenging assumptions
Classify	What assumptions . . .?	Retrospective analysis
Compare	What statement is relevant?	Reflection through journaling
Differentiate	What motive is there?	Debates
Distinguish	Related to, extraneous to, not	Discussions and other
Identify	Applicable.	Collaborating learning activities
Infer	What conclusions?	Decision-making situations
Point out	What does the author believe?	
Select	What does the author assume?	
Subdivide	Make a distinction.	
Survey	State the point of view of . . .	

What is the premise?
State the point of view of . . .
What ideas apply?
What ideas justify the conclusion?
What's the relationship between?
The least essential statements are
What's the main idea? Theme?
What inconsistencies, fallacies?
What literary form is used?
What persuasive technique?
Implicit in the statement is . .

V. **Evaluate** (according to some set of criteria, and state why)

Verbs for Objectives	**Model Questions**	**Instructional Strategies**
Appraise	What fallacies, consistencies,	Challenging assumptions
Judge	Inconsistencies appear?	Journaling
Criticize	Which is more important, moral,	Debates
Defend	better, logical, valid, appropriate?	Discussions and other
Compare	Find the errors.	collaborating learning activities
		Decision-making situations

VI. **Create (Synthesis)** (Combining elements into a pattern not clearly there before)

Verbs for Objectives	**Model Questions**	**Instructional Strategies**
Choose	How would you test . . .?	Modeling
Combine	Propose an alternative.	Challenging assumptions
Compose	Solve the following.	Reflection through journaling
Construct	How else would you . . .?	Debates
Create	State a rule.	Discussions and other
Design		collaborating learning activities
Develop		Design
Do		Decision-making situations
Formulate		
Hypothesize		
Invent		
Make		
Make Up		
Originate		
Organize		
Plan		
Produce		
Role Play		
Tell		

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