Investigation of Problem Solving in Mathematics' Teaching Learning Process

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Abstract- *Problem solving plays an important role in mathematics and should have a prominent role in mathematics education of secondary students. Many researchers believe that mathematical investigation is open and it involves both problem posing and problem solving, but some teachers have taught their students to investigate during problem solving. In this paper, we discuss the relationship between problem solving and investigation by differentiating investigation as a task, as a process and as an activity, and we show how the process of investigation can occur in problem solving.*

Keywords: Mathematics education, Problem solving, Mathematics classroom, Mathematical investigation, Secondary education.

Introduction

Nunokawa (2005) says that mathematical problem presents an objective or goal with no immediate or obvious solution or solution process. Schrock (2000) and Wilson et al. (1993) suggest that a mathematical problem must meet at least three criteria; individuals must accept an engagement with the problem, they must encounter a block and see no immediate solution process, and they must actively explore a variety of approaches to the problem. According to Cooney (1985) problem solving means different things to different people, having been viewed as a goal, process, basic skill, mode of inquiry, mathematical thinking and teaching approach. Problem solving is a process in which we perceive and resolve a gap between a present situation and a desired goal, with the path to the goal blocked by known or unknown obstacles. In general, the situation is one not previously encountered, or where at least a specific solution from past experiences is not known. Most models of problem solving include at least four phases (e.g. Bransford and Stein (1984)) a) an Input phase in which a problem is perceived and an attempt is made to understand the situation or problem; b) a Processing phase in which alternatives are generated and evaluated and a solution is selected; c) an Output phase which includes planning for and implementing the solution; and d) a Review phase in which the solution is evaluated and modifications are made, if necessary.

The call for reform in mathematics classrooms has expected teachers among other things "to create supportive learning environments, to utilise worthwhile mathematical tasks, to manage students' mathematical discourse, and to promote sense making" (Jones, (2004)). According to Guskey's (2002) model of teacher change, if teachers have been provided with relevant professional development to support these reforms, and if they have responded to the advice by changing their practice, we would expect students to have improved problemsolving outcomes (Figure 1.1). It has been proposed that one factor that has influenced the lack of adoption of problem-solving approaches has been the teachers' knowledge and beliefs about mathematics teaching and learning (Stigler and Hiebert (1999); Cai (2003); Lambdin (2003)).

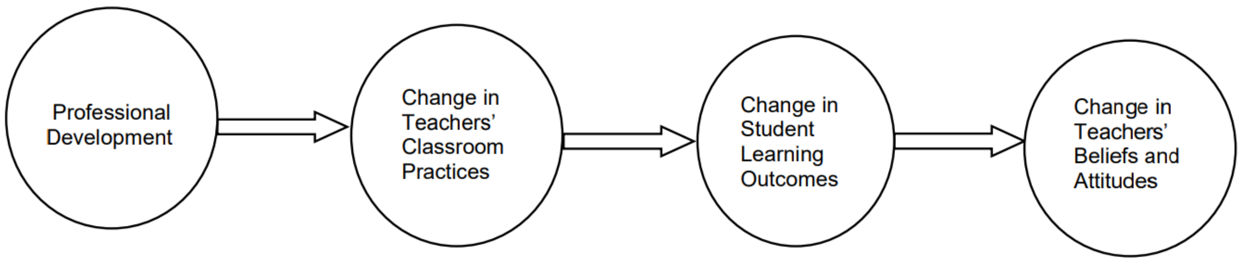

Figure1.1 A model of teacher change (Guskey, [7]).

Hiebert and Wearne (1993) refer to mathematical tasks that have the potential to provide intellectual challenges which can enhance student's mathematical development. Such tasks can promote students conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interests, curiosity. Cai and Nie (2007) recommends that students should be exposed to truly problematic tasks so that mathematical sense making is practiced. Christiansen and Walther (1986) point out that a task refers to what the teacher sets while the activity refers to what the student does in response to the task.

Task 1: Geometry (Problem-Solving Task)
In figure 1.2, segment AB is parallel to segment CD. Show that the sum of the measures of $\angle A$, $\angle E$ and $\angle C$ is 3600 . (Cai and Nie, (2007)).

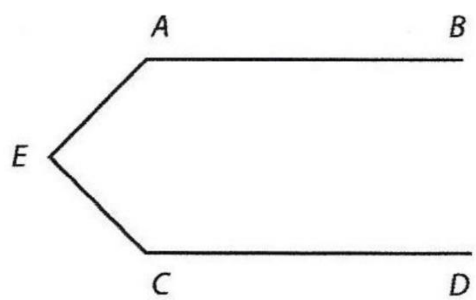

Figure1.2

This problem is found in any text book of grade 8 – 10. By making a modify the problem and ask the question what is the sum of E$\angle$A, $\angle$ and C$\angle$? Also ask the question 'find the sum of the three angles in different ways' and make generalization of the problem by asking what is the sum of the three angle measures if point E is at different location (Figure 1.3).

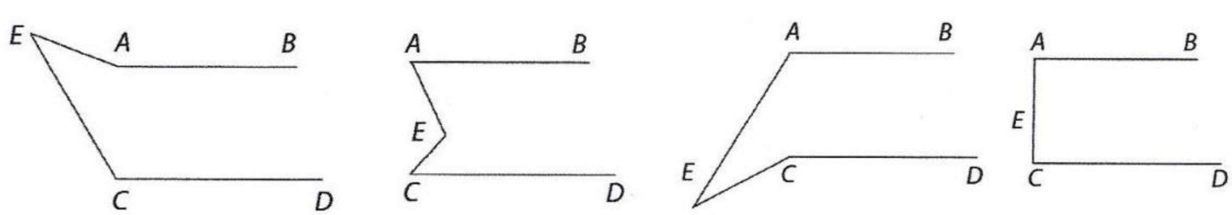

Figure1.3

This example illustrates that modifying problems that already exists in text books is often a relatively easy thing to do but increases the learning opportunity for students. Indeed, the revised problems need not be complicated or have a fancy format.

Task 2: Exponents (Open Investigative Task)
Powers of 5 are 5^1, 5^2, 5^3, 5^4, 5^5, ……… Investigate.

When students attempt an open investigative task, they are engaged in a mathematical activity which we will call an open investigative activity. Cai and Cifarelli, (2005) says that students may set a specific goal by posing a specific problem to solve but they may not have any idea what problems to pose. The latter can be called the posing of the general problem "Is there any pattern?" Both approaches can be collectively called problem posing. Therefore, an open investigative activity involves both problem posing and problem solving. In Polya's (1957) problem-solving model for closed problem-solving tasks, the first phase of understanding the problem is what a person should do before problem solving, while the actual problem-solving process begins in the second phase of devising a plan and continues into the third phase of carrying out the plan. The fourth phase of looking back is what the person should do after problem solving. But all the four phases are considered part of Polya's problem-solving model. So there is a need to differentiate between the actual process of problem solving and the entire mathematical activity of problem solving. Similarly, an open investigative activity includes what a person should do before investigation, the actual process of investigation, and what the person should do after investigation. It is clear that the first phase of understanding the task is what a person should do before investigation. Since the purpose of the second phase of problem posing is to pose problems to investigate, it seems evident that this is what the person should do before investigation. Then the third phase is the actual process of investigation. Therefore, problem posing is not part of the process of investigation although problem posing is an integral part of an open investigative activity.

Conclusion and Results

This paper recommends distinguishing between open investigative tasks, investigation as a process, and investigation as an activity involving open investigative tasks. The process of problem solving involves solving by using the process of investigation while an open investigative activity includes both problem posing and problem solving as a process. Thus investigation should not be restricted to open investigative tasks only, but it can also occur in closed problem solving tasks because investigation is primarily a process involving specialising, conjecturing, justifying and generalising (Ernest, (1991)). Hence, the characterization of mathematical investigation does not lie in the open goal of the investigative task itself, but in what it entails, i.e., the four core cognitive processes.

Knowing that investigation has nothing to do with the openness of open investigative tasks, teachers can now use closed problem-solving tasks and focus on developing the cognitive processes of mathematical investigation instead of having to sidetrack into teaching problem posing in open investigative activities. Teachers can also explain more clearly and confidently to their students what it means to investigate a problem or to engage in problem solving during an open investigative activity.

Problem Solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge and through this process; they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate and solve complex problems that require a significant amount of effort and then be encouraged to reflect on their thinking.

By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages. Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program.

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